Abstracts for the 2022 Midwest Dynamical Systems Conference

IUPUI, November 18-20

Richard Birkett (Notre Dame University)

Title: The Dynamics of Rational Skew Products

Abstract: Many useful things one takes for granted when studying the dynamics of a rational map of one complex variable become much more difficult for rational maps in complex dimension two. Here I will discuss one of these things: the relationship between the degree of a map and the degrees of its iterates. After explaining why this relationship is both difficult and important to understand, I will discuss how, in the particular case of skew products, one can sometimes cope by relating it to the dynamics of a piecewise-linear map on an infinitely branched tree.

Alexander Blokh (University of Alabama Birmingham)

Title: Moduli and their bounds

Abstract: We discuss the relation between the dynamics of rational functions and bounds of moduli of some dynamically motivated annuli.

Alex Blumenthal (Georgia Tech)

Talk cancelled.

Title: Chaotic regimes of not-small random perturbations

Abstract: The asymptotic Lyapunov exponent measures the rate at which nearby trajectories diverge; a positive exponent on a 'large' (e.g., positive volume subset) indicates strong sensitivity with respect to initial conditions, a hallmark of chaotic behavior. Despite decades of intense study, it remains a major open challenge to estimate the Lyapunov exponents of many systems of practical interest, e.g., the Chirikov standard map, for which chaotic behavior coexists and comingles with “ordered” behaviors such as sinks or elliptic islands.

In this talk, I will discuss how the introduction of noise can make this problem significantly more tractable: the scope of systems for which one can prove asymptotic chaotic behavior is greatly enhanced in this ”random” category. In this talk I will present two kinds of results: (1) direct estimates of Lyapunov exponents for random perturbations of classes of explicit 2d maps, including the Chirikov standard map; and (2) 'softer' estimates on Lyapunov exponents of more complicated systems, including random perturbations of high-dimensional ODE such as the Galerkin Navier-Stokes equations.

Marlies Gerber (Indiana University)

Title: Non-classifiability of K-automorphisms

Abstract: Within the collection of measure-preserving transformations, Bernoulli shifts have the ultimate mixing property, and K-automorphisms have the next-strongest mixing properties of any widely considered family of transformations. In particular, K-automorphisms have positive entropy and are mixing of all orders. It is known that, unlike Bernoulli shifts, the family of K-automorphisms cannot be classified up to isomorphism by a complete numerical Borel invariant. This left open the possibility of classifying K-automorphisms with a more complex type of Borel invariant. We show that this is impossible, by proving that the isomorphism equivalence relation restricted to K-automorphisms is complete analytic, and hence not Borel. This work is joint with Philipp Kunde.

Andrey Gogolyev (Ohio State University)

Title: Rigidity: from geometry to dynamics and back.

Abstract: Celebrated Otal-Croke marked length spectrum rigidity theorem recovers the geometry of a closed negatively curved surface from the periods of closed geodesics on the surface. As an intermediate step of the proof the dynamics of the (Anosov) geodesic flow is recovered from the periods. We generalize this dynamics rigidity result to the setting of volume preserving 3-dimensional Anosov flows. In turn, it leads to a more general "weighted" marked length spectrum rigidity for negatively curved surfaces. Joint work with Federico Rodriguez Hertz.

Kaitlyn Loyd (Northwestern University)

Title: A dynamical approach to the asymptotic behavior of the sequence $\Omega(n)$

Abstract: Since Birkhoff's proof of the Pointwise Ergodic Theorem, it has been shown that convergence still holds along various subsequences, such as polynomial subsequences. Bergelson and Richter recently proved that under the additional assumption of unique ergodicity, pointwise convergence holds along the number theoretic sequence $\Omega(n)$, where $\Omega(n)$ denotes the number of prime factors on n, counted with multiplicity. In this talk, we will see that removing this assumption, a pointwise ergodic theorem does not hold along $\Omega(n)$. We will then study the interplay of the dynamics with the number theoretic properties of $\Omega(n)$ to obtain further information on the asymptotic behavior of this sequence.

Vanessa Matus de la Parra (University of Rochester)

Title: Equidistribution for matings between quadratic maps and $\mathrm{PSL}(2,\mathbb{Z})$.

Abstract: In this talk, we will introduce holomorphic correspondences and the 1-parameter family studied by Bullett et al. This family lies in the gap between modularity and weak-modularity, defined by Dinh-Kauffman-Wu. Moreover, each map in the family is a "mating" between a Parabolic quadratic rational map and the modular group $\mathrm{PSL}(2,\mathbb{Z})$, and this property, together with Freire-Lopes-Mañé and Lyubich's results for rational maps on the Riemann sphere, helps us prove that we have equidistribution to both past and future, as well as periodic points.

L'ubomír Snoha (Matej Bel University)

Title: Rigidity versus flexibility for sequence entropy and polynomial entropy

Abstract: The talk is based on a joint work with Xiangdong Ye and Ruifeng Zhang, and on a joint work with Samuel Roth and Zuzana Roth.

We consider dynamical systems given by continuous selfmaps of compact metric spaces. *Flexibility* means that for a given class of dynamical systems a considered dynamical invariant can take arbitrary values, subject only to natural restrictions; flexibility as a program in dynamics was recently formulated by A. Katok. *Rigidity* in this talk means that a considered dynamical invariant can take only very restricted values for a given class of systems.

Supremum topological sequence entropy is the supremum of the topological sequence entropies of a system taken over all increasing sequences of times. *Polynomial entropy* measures the polynomial growth rate of distinguishable orbit segments. These are kinds of 'slow entropies' which can be used to distinguish between systems with zero topological entropy.

The rigidity aspect of the supremum topological sequence entropy is that it takes only values from the set $\{0, \log 2, \log 3, \dots\} \cup \{\infty\}$. In [1] we have proved the following flexibility result. Given a compact metric space X, let $S(X)$ be the set of the supremum topological sequence entropies of all continuous selfmaps of X. Then for every set $\{0\} \subseteq A \subseteq \{0, \log 2, \log 3, \dots\} \cup \{\infty\}$ there exists a one-dimensional continuum X_A with $S(X_A) = A$. In the construction of X_A we use Cook continua. This is apparently the first application of these very rigid continua in dynamics.

In [2] we show that also polynomial entropy has both rigidity and flexibility aspects. In general it is flexible – it may take any value in $[0, \infty]$, even for homeomorphisms on continua (also for continuous dendrite maps it may take many non-integer values). However, for continuous selfmaps of the interval the polynomial entropy is rigid, taking only nonnegative integer values, including infinity. To prove this, we introduce the notion of a *one-way horseshoe* and show that the polynomial entropy of an interval map equals the supremum of the lengths of one-way horseshoes of the map and its iterates (an analogue of Misiurewicz's theorem on topological entropy and standard 'two-way' horseshoes). This already implies the rigidity result. As another application

of the developed theory we compute the polynomial entropy of all maps in the logistic family.

Some open problems will be stated.

References

[1] Ľ. Snoha, X. Ye, R. Zhang, *Topology and topological sequence entropy*, Sci. China Math. **63** (2020), no. 2, 205–296.

[2] S. Roth, Z. Roth, Ľ. Snoha, *Rigidity and flexibility of polynomial entropy*, 33 pages, submitted.

Hongkun Zhang (University of Massachusetts Amherst)

Title: Hyperbolicity of certain chaotic billiards

Abstract: Billiards are dynamical systems generated by a straight-line motion of a point particle within a domain with a piecewise smooth boundary. Upon reaching the boundary of the domain (billiard table), the billiard particle reflects according to the elastic collision law. They are well-known models first introduced by Birkhoff as paradigmatic examples of Hamiltonian systems, and pioneered by Yakov Sinai, Leonid Bunimovich, Pavel Bleher, etc., as a mathematical model for the Lorenz systems and hard-ball gases. Since then billiards have acquired increasing importance as they shed light in understanding thermodynamic limits, connected to deep issues in quantum and wave physics all the way to quantum chaos. However for some Bunimovich billiards, whose boundary consists of only arcs and straight lines, even the hyperbolicity is not known, not to mention ergodicity and other chaotic properties. In this talk, I will discuss these new classes of convex billiards and their properties. The hyperbolicity for some of them is only numerically proved, which leaves many open questions to explore. I will also review my recent collaboration work with Michal Misiurewicz about topological entropy for Bunimovich stadium.