Pre - Chapter 9

Matrix Multiplications Notes

The following example will be helpful in Markov Chain section (Section 9.2).

If: $A = \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix}$ find A^2, A^3, A^4 and A^5

$$A^2 = A.A = \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} . \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix}$$

$$A^3 = A^2.A = \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix} . \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = \begin{vmatrix} -3 & 1 \\ -2 & -2 \end{vmatrix}$$

$$A^4 = A^2.A^2 = \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix} . \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 3 \\ -6 & 2 \end{vmatrix}$$

$$A^5 = A^2.A^3 = \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix} . \begin{vmatrix} -3 & 1 \\ -2 & -2 \end{vmatrix} = \begin{vmatrix} 5 & 1 \\ -2 & 6 \end{vmatrix}$$

Examples to be solved before chapter 9 (*strongly recommended*)

1) If: $T = \begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix}$. Find:

a) T^2 b) T^3 c) T^4 d) T^5

Answers: 1) *a)* $\begin{bmatrix} 0.520 & 0.480 \\ 0.360 & 0.640 \end{bmatrix}$ *b)* $\begin{bmatrix} 0.392 & 0.608 \\ 0.456 & 0.544 \end{bmatrix}$ *c)* $\begin{bmatrix} 0.433 & 0.557 \\ 0.418 & 0.582 \end{bmatrix}$ *d)* $\begin{bmatrix} 0.423 & 0.577 \\ 0.433 & 0.567 \end{bmatrix}$

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2) If: $P = \begin{vmatrix} 0.3 & 0.7 \end{vmatrix}$ and $T = \begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix}$. Use the results of question 1 to multiply:

a) $P.\ T$ b) $P.\ T^2$ c) $P.\ T^3$ d) $P.\ T^4$

Answers: 2) *a)* $\begin{vmatrix} 0.48 & 0.52 \end{vmatrix}$ *b)* $\begin{vmatrix} 0.408 & 0.592 \end{vmatrix}$ *c)* $\begin{vmatrix} 0.4368 & 0.5632 \end{vmatrix}$ *d)* $\begin{vmatrix} 0.4253 & 0.5747 \end{vmatrix}$

3) If: $T = \begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0.2 & 0.4 & 0.4 \\ 0 & 0.1 & 0.9 \end{bmatrix}$. Find:

a) T^2 b) T^3 c) T^4

Answers: 3) *a)* $\begin{bmatrix} 0.07 & 0.21 & 0.72 \\ 0.1 & 0.26 & 0.64 \\ 0.02 & 0.13 & 0.85 \end{bmatrix}$ *b)* $\begin{bmatrix} 0.049 & 0.177 & 0.774 \\ 0.062 & 0.198 & 0.740 \\ 0.028 & 0.143 & 0.829 \end{bmatrix}$ *c)* $\begin{bmatrix} 0.040 & 0.163 & 0.797 \\ 0.046 & 0.172 & 0.782 \\ 0.031 & 0.149 & 0.820 \end{bmatrix}$

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4) If: $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \end{bmatrix}$ and $T = \begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0.2 & 0.4 & 0.4 \\ 0 & 0.1 & 0.9 \end{bmatrix}$ Use the results of question 3 to multiply:

a) $P.\ T$ b) $P.\ T^2$ c) $P.\ T^3$ d) $P.\ T^4$

Answers: 4) *a)* $\begin{bmatrix} 0.08 & 0.23 & 0.69 \end{bmatrix}$ *b)* $\begin{bmatrix} 0.054 & 0.185 & 0.761 \end{bmatrix}$

c) $\begin{bmatrix} 0.0424 & 0.1663 & 0.7913 \end{bmatrix}$ *d)* $\begin{bmatrix} 0.0375 & 0.15837 & 0.80413 \end{bmatrix}$