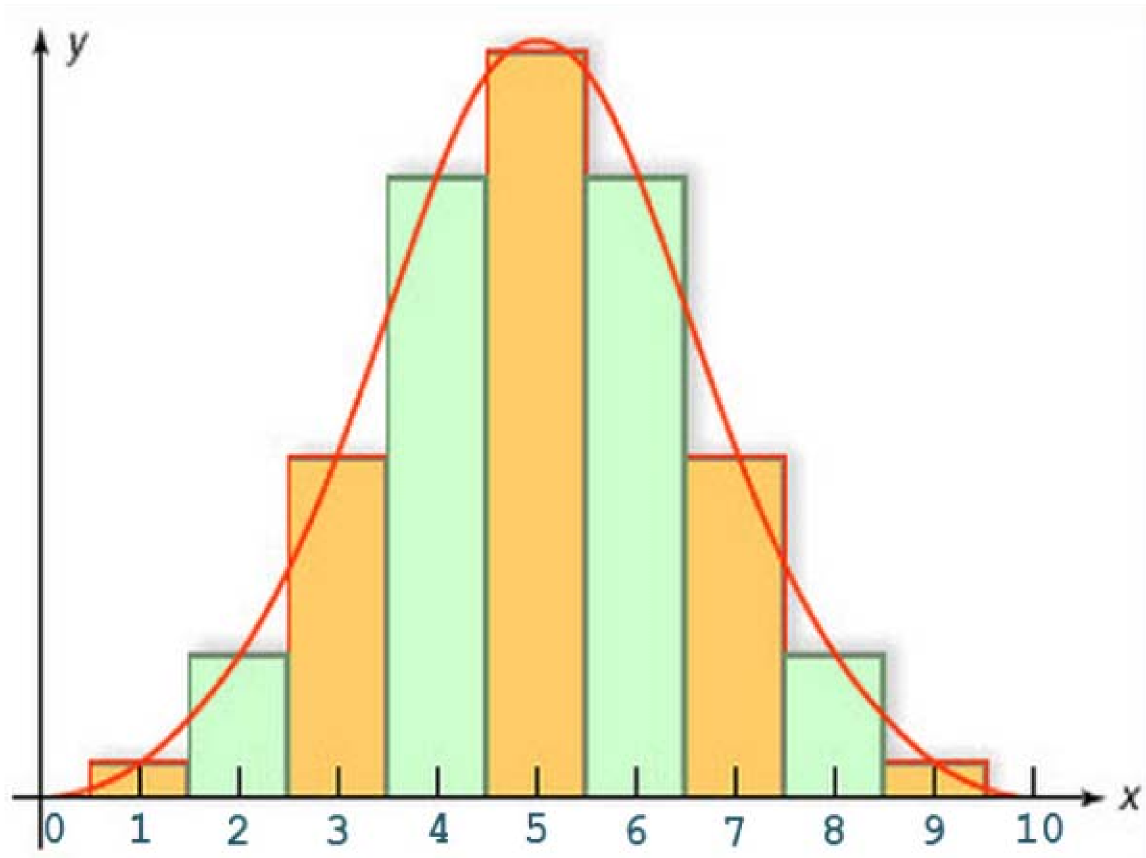
**Section 5.4: Normal Approximation To The Binomial**



**<u>RULES:</u>** To approximate **<u>binomial</u>** probability by normal curve area:

**Step 1)** determine $n, p, q$

**Step 2)** check that both $n.p > 5$ <u>and</u> $n.q > 5$

**Step 3)** find the expected value and the standard deviation

$$\mu = n \cdot p \qquad\qquad \sigma = \sqrt{n.p.q}$$

**Step 4)** find the new points by:

* subtracting 0.5 from the starting point
* adding 0.5 to the finish point

*examples:*

| | | |
|---|---|---|
| $P(3 \leq X \leq 6)$ | *will be* | $P(2.5 \leq X \leq 6.5)$ |
| $P(X = 7)$ | *will be* | $P(6.5 \leq X \leq 7.5)$ |
| $P(X \geq 8)$ | *will be* | $P(X \geq 7.5)$ |
| $P(X \leq 8)$ | *will be* | $P(X \leq 8.5)$ |

**Step 5)** find the Z-scores and the area under the normal curve using the table

**Example 1**: According to the Department of Health and Human Services, the probability is about 80% that a person aged 70 will be alive at the age of 75. Suppose that 500 people aged 70 are selected at random. Find the probability that:

a) exactly 390 of them will be alive at the age of 75

**a)** Step 1) $n = 500,\quad p = 0.8,\quad q = 0.2$

Step 2) check if both $n.p$ and $n.q$ are more than 5:

$$n.p = (500).(0.8) = 400$$

$$n.q = (500).(0.2) = 100$$

Step 3) find the expected value and the std. deviation:

$$\mu = n \cdot p = (500).(0.8) = 400$$

$$\sigma = \sqrt{n.p.q} = \sqrt{(500).(0.8).(0.2)} = 8.94$$

Step 4) find the new point:

$$P(X = 390) \quad \text{will be} \quad P(389.5 \leq X \leq 390.5)$$

Step 5) find the Z-score:

$$X = 389.5, \qquad Z = \frac{389.5 - 400}{8.94} = -1.17$$

$$X = 390.5, \qquad Z = \frac{390.5 - 400}{8.94} = -1.06$$

and now by using the table:

$$P(-1.17 \leq Z \leq -1.06) = 0.1446 - 0.1210 = 0.0236$$

**Example 1 (Cont.)** : According to the Department of Health and Human Services, the probability is about 80% that a person aged 70 will be alive at the age of 75. Suppose that 500 people aged 70 are selected at random. Find the probability that:

**b)** for $P(375 \le X \le 425)$, we use the information of steps 1, 2 and 3 then:

$P(375 \le X \le 425)$ will be $P(374.5 \le X \le 425.5)$

$$X = 374.5, \qquad Z = \frac{374.5 - 400}{8.94} = -2.85$$

$$X = 425.5, \qquad Z = \frac{425.5 - 400}{8.94} = 2.85$$

and now by using the table:

$P(-2.85 \le Z \le 2.85) = 0.9978 - .0022 = 0.9956$

| | **Section 5.3** No Approximation | **Section 5.4** Approximation |
|---|---|---|
| **Given** | Expected value<br>Standard Deviation | $n, p$ |
| **Steps** | • Find: Z-Score: $Z = \frac{X-\mu}{\sigma}$<br>• Use the table | • Find: $q$ *where* $q = 1 - p$<br>excepted value $E[X] = \mu = n.p$<br>Standard deviation $\sigma = \sqrt{n.p.q}$<br>• Add / subtract 0.5 as needed<br>• Find the Z-Score : $Z = \frac{X-\mu}{\sigma}$<br>• Use the table |

**Example 2:** A coin with Pr[Tails] = 0.4 is flipped 200 times. Find the probability of getting between 65 and 100 tails on the coin. Give your answer as a decimal number correct to three decimal places

**Example 3:** Assume that IQ scores are normally distributed with mean 100 and standard deviation 15. What is the probability that a randomly chosen person will have an IQ at most 105?