Section 5.2 Expected Value and Standard Deviation

Random Variable: A function X that assigns to every outcome exactly one real number.

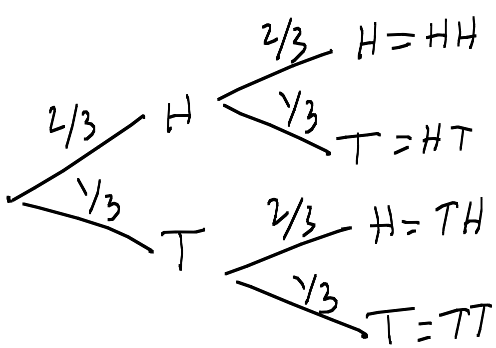
Probability Density Function: A list of all possible values of the random variables and the associated probabilities.

Outcomes (events)	Random Variable (X)	Prob. Density (P)
all possibilities	value of each possibility	prob. of each possibility
		Sum = 1

Example 1: An unfair coin in which P(H) = 2/3 is flipped twice. The random variable X is defined to be the number of heads. Find the density function.

$P(T) = 1/3$

X = the number of heads



Outcomes	X	P
HH	2	2/3·2/3 = 4/9
HT or TH	1	2/3·1/3 + 1/3·2/3 = 4/9
TT	0	1/3·1/3 = 1/9

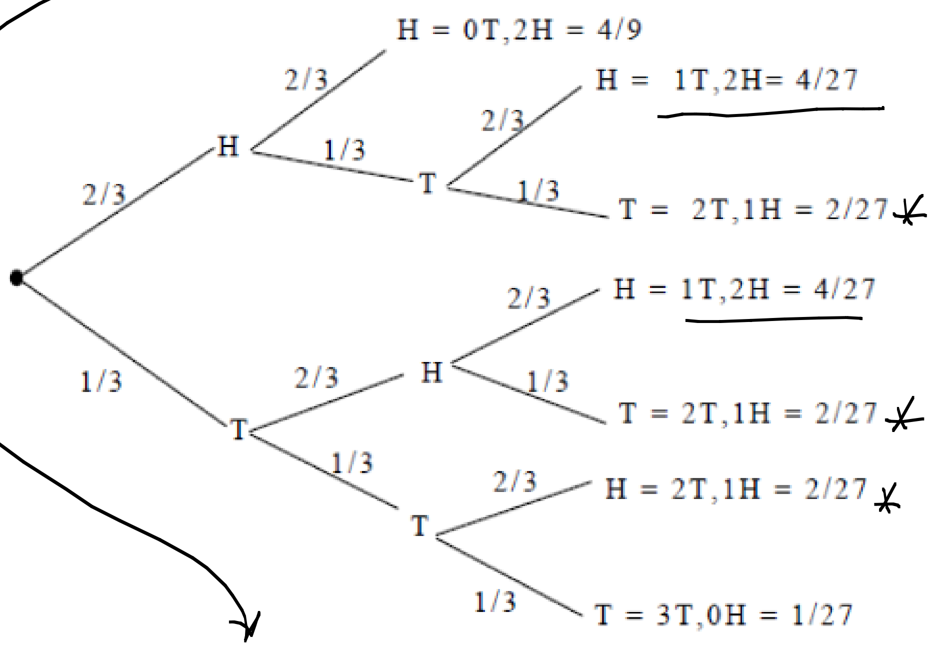
= 1

***Using the Tree**: Use it when the problem is written is way that the experience stops when certain condition is met. See the next example and how the word "until" is an indication of tree is needed.*

Example 2: An experiment consists of flipping an unfair coin where $P(H) = 2/3$ **until** a total of 2 heads occur or 3 flips. The random variable is defined to be the number of tails. Find the expected value of the random variable

$P(H) = 2/3$

$P(T) = 1/3$



Outcomes	X	P	X.P
0T, 2 H	X_1 0	P_1 4/9	0
1T, 2 H	X_2 1	P_2 4/27 + 4/27 = 8/27	8/27
2T, 1 H	X_3 2	P_3 2/27 + 2/27 + 2/27 = 6/27	12/27
3T, 0 H	X_4 3	P_4 1/27	3/27
	Sum	= 1	$E(X)$ = 23/27 = 0.85

$E(x)$

μ

Binomial & Non-Binomial Distribution *(Using Tables)*

- **Expected Value E[X], μ:** The mean, the average value of a random variable in which:
  $\mu = E[X] = \sum P_i X_i = X_1P_1 + X_2P_2 + X_3P_3 + \ldots\ldots$
- **Variance** σ^2 **:** $\sigma^2 = \sum P_i(X_i - \mu)^2$ **The Variance is a measure of the dispersion of the distribution of a random variable.**
- **Standard Deviation:** $\sigma = \sqrt{\sigma^2}$

Probability Types (from chapter 4):

1) Binomial Distribution: The probability of section 4.4 or Bernoulli trials:

(repeated events) is applied and the probability of the repeated events **is the same**:

Common example: flipping coins, or when applying same given probability on all selected parts. See example 3

2) Non Binomial Distribution: The probability of section 4.1:

$$\text{Probability} = \frac{\text{Number of choices of what we are looking for}}{\text{Number of All possible choices}}$$

Common example: Selecting team of people, cards when the probabilty changes (first card is out of 52, second is out of 51 and so on) See example 5.

Example 3: Stereo speakers manufactured with probability of 20% being defective. Three are selected off continuous assembly line, define the random variable X as the number of the defective parts. Find:

a) the density function and the expected value for the defective parts

b) the expected value for the good parts

c) the variance, the standard deviation.

$\boxed{n = 3,}$ $P(D) = 0.2,$ $P(G) = 0.8$

a) *X = number of defective parts*

Outcomes	X	P	$X.P$
0D, 3G	0	$C(3,0).(0.80)^3.(0.20)^0 = 0.51$	0
1D, 2G	1	$C(3,1).(0.80)^2.(0.20)^1 = 0.38$	0.38
2D, 1G	2	$C(3,2).(\underset{G}{0.80})^1.(\underset{D}{0.20})^2 = 0.096$	0.192
3D, 0G	3	$C(3,3).(0.80)^0.(0.20)^3 = 0.008$	0.024
		Sum = 1	= 0.6

Expected value for the defective part is = 0.6

$\mu_D = 0.6$

$\mu_G = 2.4$

b) Expected value for the good part is = 3 - 0.6 = 2.4

Example 3Cont.:

Def

$\mu = 0.6$

X	P
0	$C(3,0).(0.80)^3.(0.20)^0 = 0.51$
1	$C(3,1).(0.80)^2.(0.20)^1 = 0.38$
2	$C(3,2).(0.80)^1.(0.20)^2 = 0.096$
3	$C(3,3).(0.80)^0.(0.20)^3 = 0.008$

c)

X	P	$(X_i - \mu)$	$(X_i - \mu)^2$	$P_i(X_i - \mu)^2$
0	0.51	(0 -0.6) = -0.6	$(-0.6)^2 = 0.36$	0.51(0.36)= 0.184
1	0.38	(1 -0.6) = 0.4	$(0.4)^2 = 0.16$	0.38 (0.16)= 0.061
2	0.096	(2 -0.6) = 1.4	$(1.4)^2 = 1.96$	0.096 (1.96)= 0.19
3	0.008	(3 -0.6) = 2.4	$(2.4)^2 = 5.76$	0.008 (5.76)= 0.05
			Sum =	0.48

σ^2

Variance σ^2: $\sigma^2 = \sum P_i(X_i - \mu)^2 = 0.48$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.48} = 0.69$

Binomial Distribution *(Using Formula)*

No Tree Required

- **Expected Value** $E[X]$ or μ where : $E[X] = \mu = n \cdot p$
- **Variance** σ^2 where $\sigma^2 = n \cdot p \cdot q$

5.3, 5.4

- **Standard Deviation:** $\sigma = \sqrt{\sigma^2}$

Example 4: Solve example 3 again but without table

This problem is Binomial, first find *n,p & q:* $n = 3$, $p = 0.2$, $q = 0.8$

Expected Value: $\mu = n \cdot p = 3(0.2) = 0.6$ for the defective parts

Variance: $\sigma^2 = n \cdot p \cdot q = 3(0.2)(0.8) = 0.48$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.48} = 0.69$

Example 5: A box with 6 good parts and 4 defective in which 3 are selected. The random variable X is defined as the number of defective parts selected. Find:

a) the density function and the expected value for the defective parts
b) the expected value for the good parts
c) the variance, the standard deviation.

6 G
4 D

10 Parts
3 Selected

This problem is **not Binomial**.

$P = \frac{\quad}{C(10,3)}$

X = *number of defective parts*

Outcomes	X	P	$X.P$
0D,3G	0	$\frac{C(4,0).C(6,3)}{C(10,3)} = \frac{20}{120}$	0
1D,2G	1	$\frac{C(4,1).C(6,2)}{C(10,3)} = \frac{60}{120}$	$\frac{60}{120}$
2D,1G	2	$\frac{C(4,2).C(6,1)}{C(10,3)} = \frac{36}{120}$	$\frac{72}{120}$
3D,0G	3	$\frac{C(4,3).C(6,0)}{C(10,3)} = \frac{4}{120}$	$\frac{12}{120}$
	Sum	= 1	= 1.2

$\mu_D = 1.2$

Expected value for the defective part is = 1.2

b) Expected value for the good part is = 3 - 1.2 = 1.8

Example 5 Cont.:

X	P
0	$\frac{C(4,0).C(6,3)}{C(10,3)}=\frac{20}{120}$
1	$\frac{C(4,1).C(6,2)}{C(10,3)}=\frac{60}{120}$
2	$\frac{C(4,2).C(6,1)}{C(10,3)}=\frac{36}{120}$
3	$\frac{C(4,3).C(6,0)}{C(10,3)}=\frac{4}{120}$

$\mu = 1.2$

c)

X	P	$(X_i-\mu)$	$(X_i-\mu)^2$	$P_i(X_i-\mu)^2$
0	$\frac{20}{120}$	(0 -1.2) = -1.2	$(-1.2)^2$=1.44	$\frac{20}{120}$(1.44)= 0.24
1	$\frac{60}{120}$	(1 -1.2) = -0.2	$(-0.2)^2$=0.44	$\frac{60}{120}$ (0.44)= 0.02
2	$\frac{36}{120}$	(2 -1.2) = 0.8	$(0.8)^2$=0.64	$\frac{36}{120}$ (0.64)= 0.192
3	$\frac{4}{120}$	(3 -1.2) = 1.8	$(1.8)^2$=3.24	$\frac{4}{120}$ (3.24)= 0.108
			Sum =	0.56

$\sigma^2 = 0.56$

Variance σ^2: $\sigma^2 = \sum P_i(X_i-\mu)^2 = 0.56$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.56} = 0.748$

Example 6: A multiple-choice test contains 10 questions with 4 choices for each answer. If a student guesses the answers, find:

a) the probability that he will get 4 correct answers.

b) the expected value for the correct answers

c) the expected value for the wrong answers

d) the variance, the standard deviation.

$n = 10$

$P(c) = 1/4, \quad q = 3/4$

This problem is a Binomial, find n, p & q: $n = 10$, $p = 1/4 = 0.25$, $q = 0.75$.

a) $P = C(10,4).\underset{C}{(0.25)^4}.\underset{W}{(0.75)^6}$

b) Expected Value: $\mu = n \cdot p = 10(0.25) = \underline{\underline{2.5}}$ for the correct answers

c) Expected Value: $\mu = n \cdot p = 10(0.75) = \underline{\underline{7.5}}$ for the wrong answers

d) Variance: $\sigma^2 = n \cdot p \cdot q = 10(0.25)(0.75) = 1.875$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{1.875} = 1.37$

Example 7: Two coins are selected at random from a pocket that contains 2 nickels and 6 quarters. The random variable X is the total value in cents of the 2 selected coins. Find E(X).

2N, 6Q

Two are Selected

outcomes	X	$P = \frac{\quad}{C(8,2)}$	$X \cdot P$
2 N	10	$P = \frac{C(2,2)}{C(8,2)} = \frac{1}{28}$	$\frac{10}{28}$
2 Q	50	$P = \frac{C(6,2)}{C(8,2)} = \frac{15}{28}$	$\frac{750}{28}$
1N+1Q	30	$P = \frac{C(2,1) \cdot C(6,1)}{C(8,2)} = \frac{12}{28}$	$\frac{360}{28}$

$\mu = 40¢$

<u>Example 8:</u> By rolling a pair of dice, a game is played in which:

You win \$2 if the sum is 2, 3, 4 or 5.

You win \$3 if the sum is 6, 7 or 8.

You loose \$5 if the sum is 9, 10, 11 or 12.

If you pay \$2 to play the game, find the expect gain or loss.

23
32
41
14
} Sum = 5

Outcomes	X	P	X.P
2, 3, 4, 5	+2	1/36 + 2/36 + 3/36 + 4/36 = 10/36	20/36
6, 7, 8	+3	5/36 + 6/36 + 5/36 = 16/36	48/36
9, 10, 11, 12	-5	4/36 + 3/36 + 2/36 + 1/35 = 10/36	-50/36
	Sum	1	E(X)=\$ 0.5

Expected gain is \$0.5, but you paid \$2 to play the game, then there is a loss of \$1.5