Chapter 4

Section 4.4: Bernoulli Trials

$P(W) = 0.6$
$P(L) = 0.4$

Example 1: The probability that a team will win a game is 60%. Find the probability that:

a) the team wins a game out of 2

W.L or LW

$(0.6)\cdot(0.4) + (0.4)(0.6) = 2(0.6)(0.4)$

b) the team wins the first 2 games out of 3

$WWL \longrightarrow P = (0.6)(0.6)(0.4) = (0.6)^2(0.4)$

c) the team wins 2 game out of 3

WWL or WLW or LWW

$P = 3(0.6)^2\cdot(0.4)^1$

d) the team wins 2 games out of 4

(WWLL) or (WLWL) or (WLLW) or (LLWW) or (LWLW) or (LWWL)

$P = 6\cdot\underset{W}{(0.6)^2}\cdot\underset{L}{(0.4)^2}$

$P = C(4,2)\underset{W}{(0.6)^2}\underset{L}{(0.4)^2}$

<u>Bernoulli trial:</u> (repeated events) is applied when:

1) each event has two outcomes only, (win, loose); (pass, fail)...

2) the sum of the two probabilities for the two outcomes is = 1

3) the events are independent

4) the probability in the repeated events is the same

$$\mathbf{P = C(\mathit{n}\ ,\ \mathit{r})\ .\ \mathit{p}^{\mathit{r}}.\ \mathit{q}^{\mathit{n-r}}} \qquad \mathbf{(\mathit{q}\ = 1 - \mathit{p})}$$

p : probability of success *(what we are looking for)*

n : total number of trials

r : number of successes *(number of events of what we are looking for)*

Example 2: the probability of winning a game is 60%. If the team plays 8 games, find the probability that the team wins:

a) 5 games   b) at least 6 games   c) at least 2 games

$P(W) = 0.6, \quad P(L) = 0.4, \quad n = 8$

a) 5W out of 8 $\longrightarrow C(8,5)\underset{W}{(0.6)^5}\underset{L}{(0.4)^3}$

$= 56 \cdot (0.0777) \cdot (0.064) = 0.27869$

$\approx 27.9\%$

b) 6W or 7W or 8W

$C(8,6)\underset{W}{(0.6)^6}(0.4)^2 + C(8,7)(0.6)^7(0.4)^1 + C(8,8)(0.6)^8(0.4)^0$

$= 0.209 + 0.0896 + 0.0168$

$= 0.3154 \approx 31.54\%$

c) 0W or 1W or $\underbrace{2W \text{ or } 3W \text{ or } 4W \text{ or } 5W - \text{ or } 8W}_{\text{at least } 2W}$

$= P(\text{all}) - P(0W) - P(1W)$

$= 1 - C(8,0)\underset{W}{(0.6)^0}(0.4)^8 - C(8,1)(0.6)^1(0.4)^7$

$= 1 - 0.007 - 0.079 = 0.9915$

$\approx 99.15\%$

Example 3: By taking a test of 10 questions, each question has 4 choices for an answer and only one answer is correct. If a student is answering the questions by guessing, find the probability that he gets at least 2 correct questions

$n = 10, \quad P(C) = \frac{1}{4} = 0.25, \quad q = 0.75$

P(at least 2C out of 10)

$0C \text{ or } 1C \text{ or } \underbrace{2C \text{ or } 3C \text{ or } 4C \text{ or } 5C \ldots \ldots 10C}_{\text{at least } 2C}$

$\rightarrow P = P(\text{all}) - P(0C) - P(1C)$

$= 1 - C(10,0)(\underset{C}{0.25})^0(\underset{W}{0.75})^{10} - C(10,1)(0.25)^1(0.75)^9$

$= 1 - 0.0563 - 0.1877 = 75.60\%$

$\rightarrow P = P(\text{all}) - P(10W) - P(9W)$

$= 1 - C(10,10)(\underset{C}{0.25})^0(\underset{W}{0.75})^{10} - C(10,9)(0.25)^1(0.75)^9$

$= 75.60\%$