Chapter 4

Section 4.3: Bayes Theorem

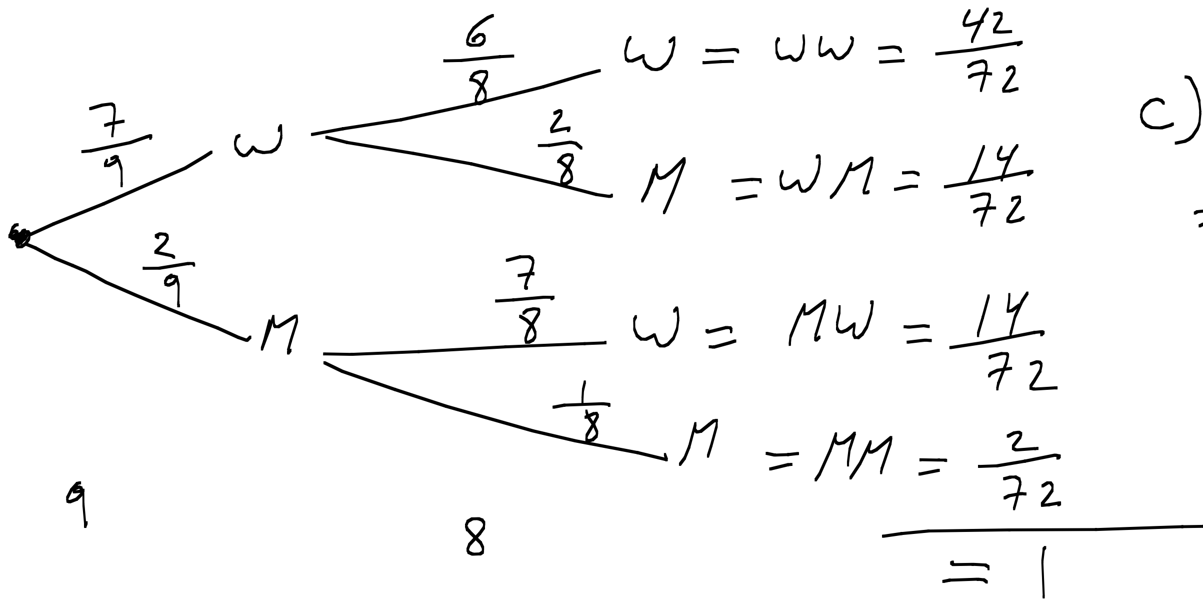
Use the tree method when you have an experiment which consists of a sequence of sub-experiments.

Example 1: Two people will be selected without replacement out of 7 women and 2 men.

a) draw the tree and show all the probabilities.

b) find the probability that 2 women are selected

c) find the probability that 2 of the same gender are selected



b) $P(2W) = \frac{42}{72}$

c) $P(2W \text{ or } 2M)$

$$= \frac{42}{72} + \frac{2}{72}$$

$$= \frac{44}{72}$$

NOTE: By using the tree method:

a) the sum of all path probabilities must be = 1

b) the sum of probabilities of all branches from one node = 1

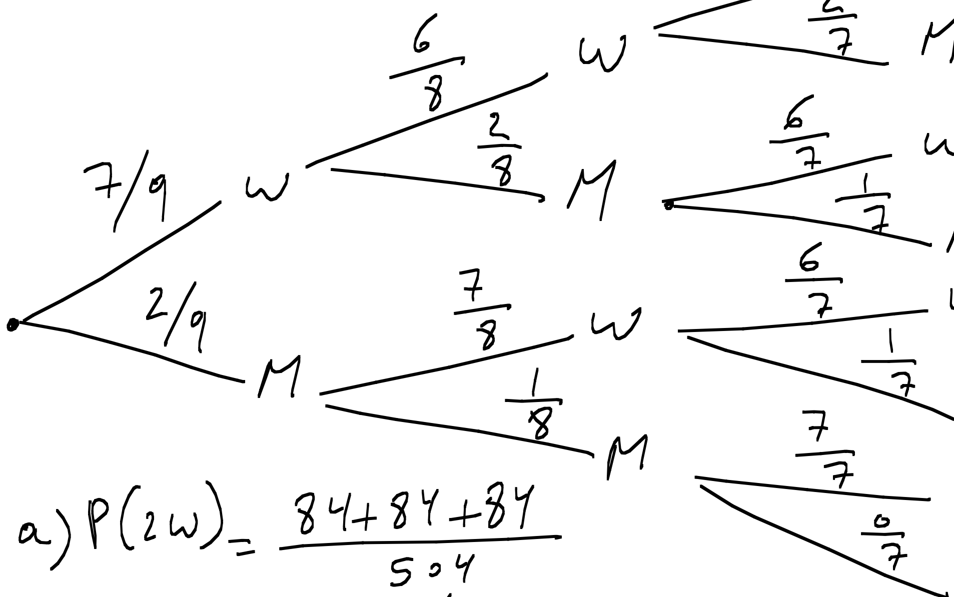
c) the sum of all path probabilities that branches from a given node must be equal to the probability reaching that node.

Example 2: Repeat example 1 but this time 3 people are selected without replacement, and find the probability of:

a) exactly 2 are women

b) at least 2 are women

c) 1 man given that at least 2 are women



$W = WWW = \frac{210}{504}$

$M = WWM = \frac{84}{504}$ ←

$W = WMW = 84/504$ ←

$M = WMM = 14/504$

$W = MWW = 84/504$ ←

$M = MWM = 14/504$

$W = MMW = 14/504$

$M = \phi$ $= 1$

a) $P(2W) = \frac{84+84+84}{504}$

$= 252/504$

b) $P(\text{at least } 2W) = \frac{252}{504} + \frac{210}{504} = \frac{462}{504}$

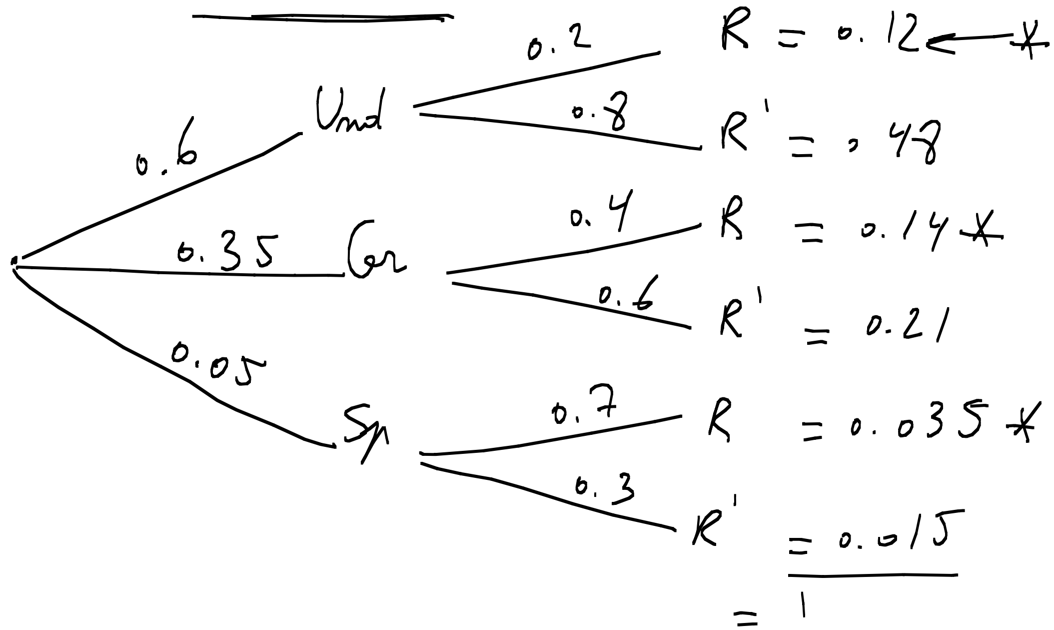
c) $P(1M \, \& \, 2W \mid \text{at least } 2W) = \frac{252}{462}$

Example 3: At a state university, 60% are undergraduates, 35% graduates and 5% are in special program. Also, 20% of the undergraduates are married, 40% of the graduates are married and 70% of the special program are married. Draw the tree and find the following probabilities that:

a) a selected student is married and undergraduate

b) a selected student is married

c) a selected married student is an undergraduate



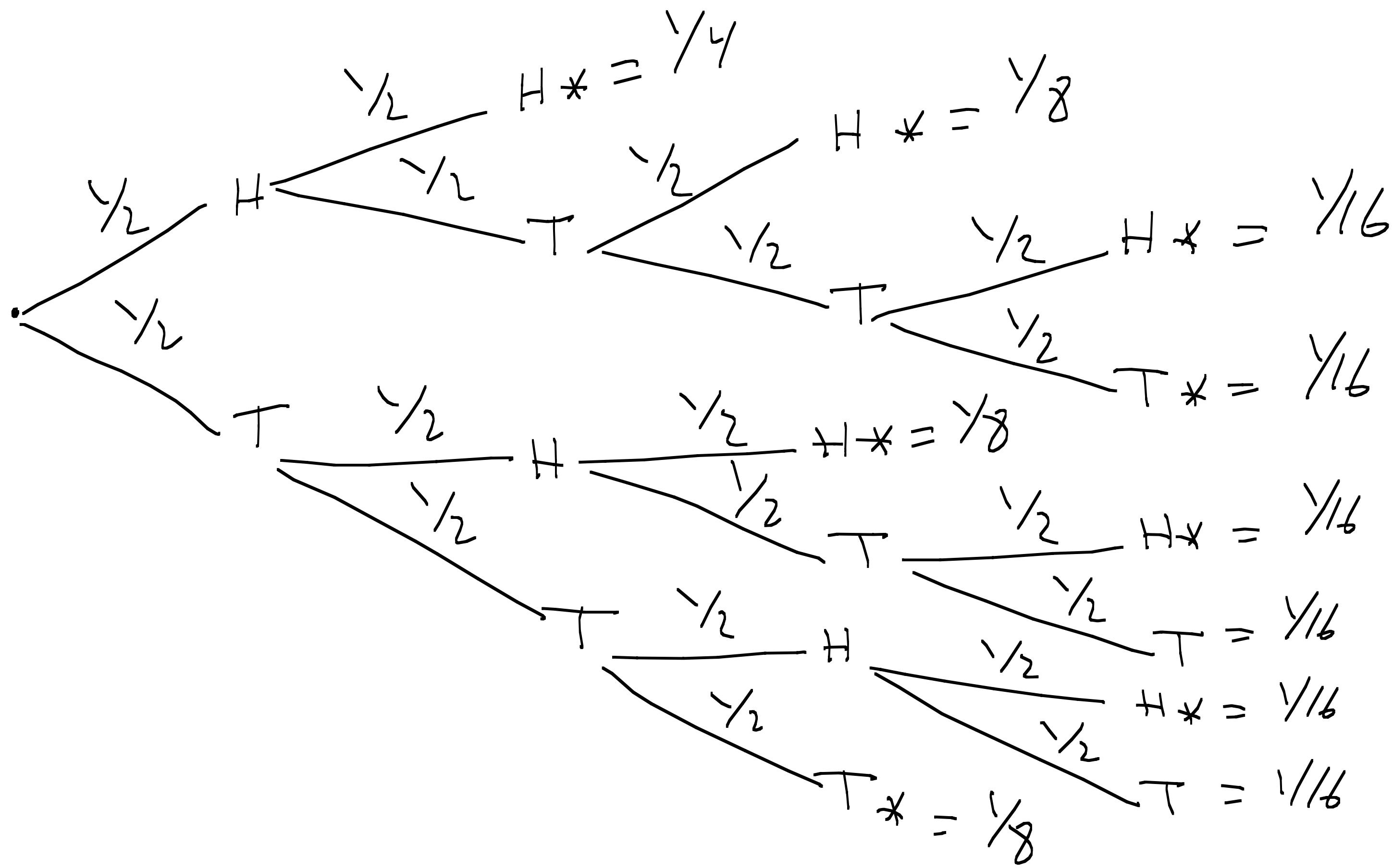
a) $P = 0.12$

b) $P(R) = 0.12 + 0.14 + 0.035 = 0.295$

c) $P(\text{Und} \mid R) = \dfrac{0.12}{P(R)} = \dfrac{0.12}{0.295} \approx 0.41$

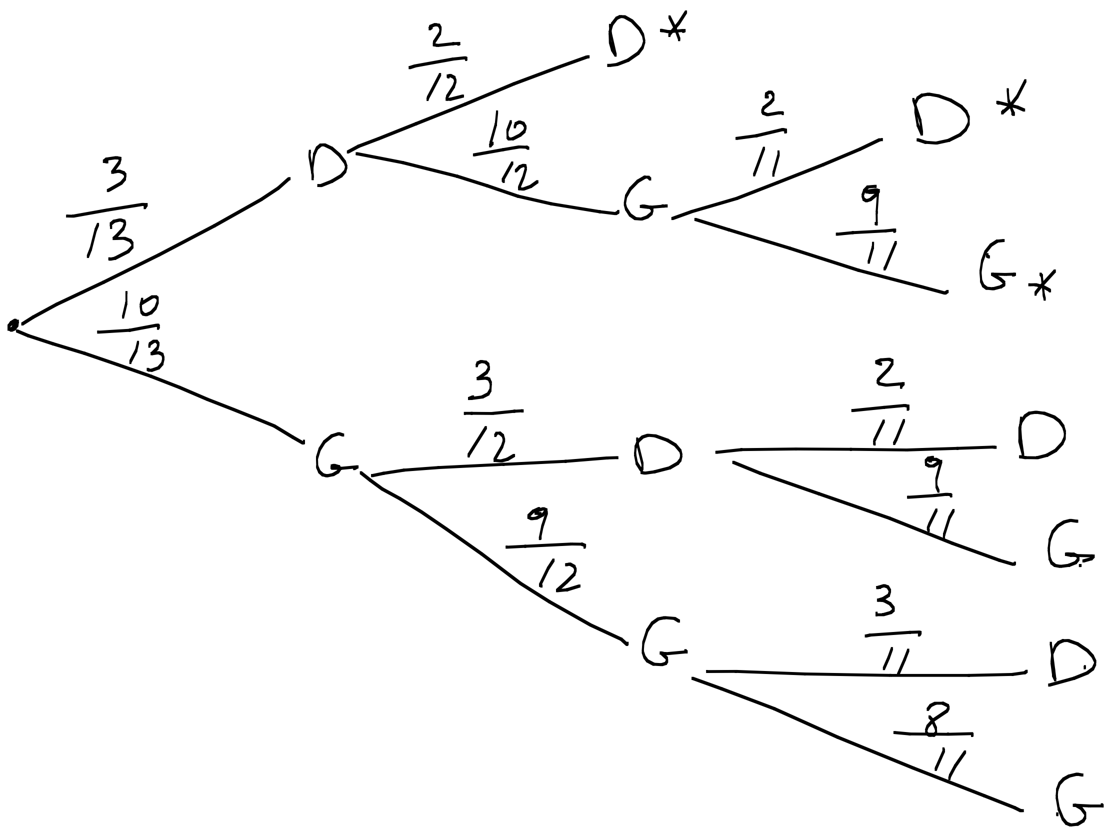
Example 4: A fair coin is flipped until 2 heads or 3 tails appear. Draw a tree and determine all probabilites

$P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$ 2 H or 3 T



Example 5: A box contains 10 good parts and 3 defective parts, if parts are selected without replacement one after another until either 2 defective parts are found or three are selected. Draw the tree and show the probabilities

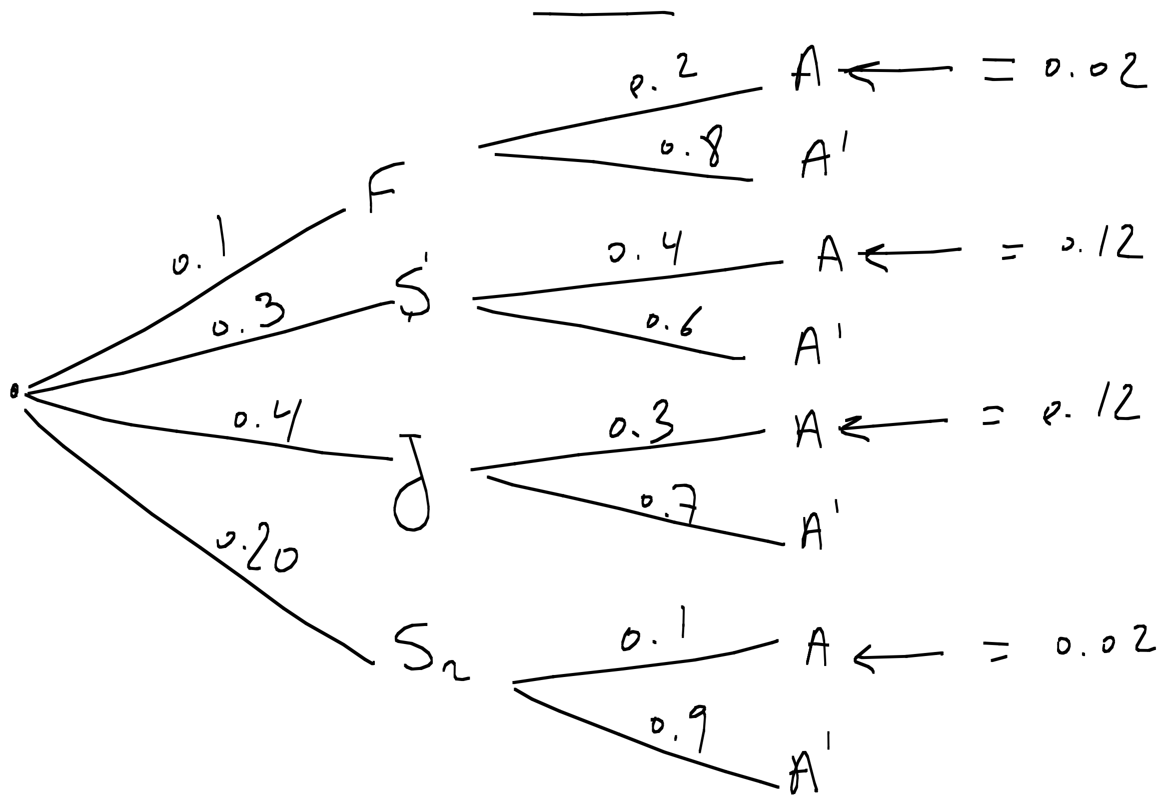
$$\underbrace{10G,\ 3D}_{13}$$



Example 6: In a certain class, there were 10% freshman, 30% sophomores, 40% juniors and 20% seniors. Past experiences show that 20% of freshmen , 40% of sophomores, 30% of juniors and 10% of seniors get A. If one student was selected at random:

a) find the probability that this student got an A

b) if the student found to be an A student, find the probability that this student was a junior



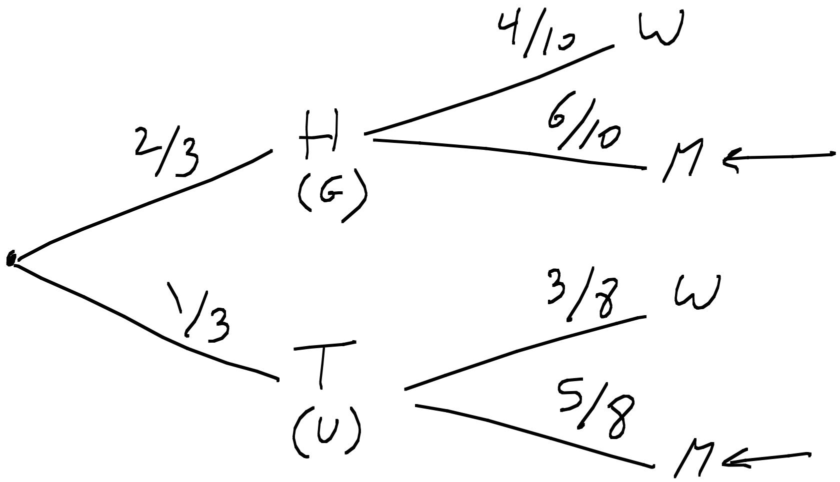
a) $P(A) = 0.02 + 0.12 + 0.12 + 0.02 = 0.28$

b) $P(J|A) = \frac{0.12}{0.28} = 0.43$

Example 7: Two groups of students applied for a job, graduate group (4 women and 6 men) and undergraduate group (3 women and 5 men). The company would flip an unfair coin in which P(H) = 2/3, if it is a head then the graduate group will be selected and a student from that group will be selected.

a) find the probability that a man is selected

b) if the person selected was a man, find the probability that he is from the undergraduate group



a) $P(M) =$

$$\frac{2}{3}\cdot\frac{6}{10} + \frac{1}{3}\cdot\frac{5}{8}$$

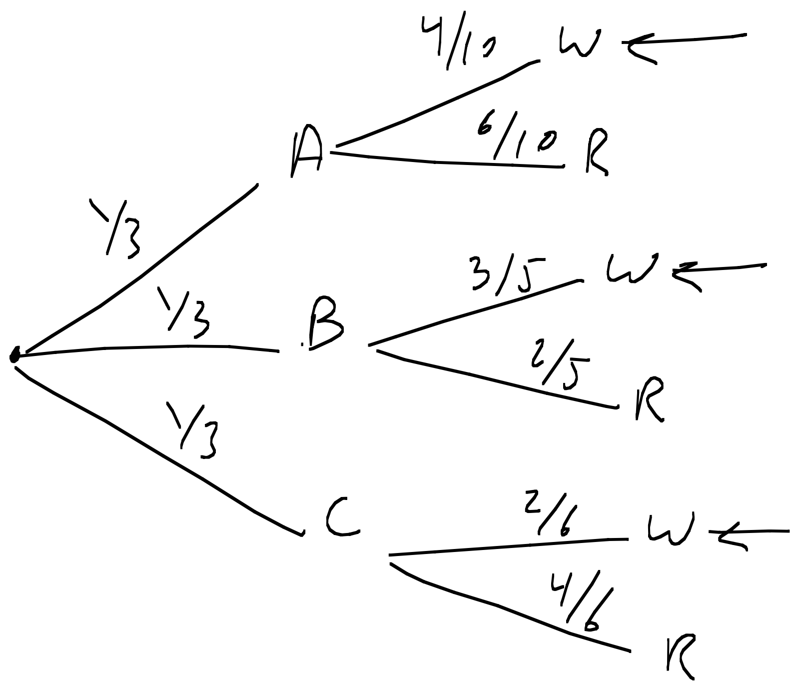
$$= \frac{73}{120} \approx 0.61$$

b) $$P(U|M) = \frac{\frac{1}{3}\cdot\frac{5}{8}}{0.61} = \frac{0.21}{0.61} \approx 0.34$$

Example 8: Box A contains 4 white books and 6 red books.
Box B contains 3 white books and 2 red books.
Box C contains 2 white books and 4 red books.

a) if a box was selected and <u>then</u> a book was selected, what is the probability that this book is white

b) if the book selected was white, what is the probability that this book was from box B?



a) $P(W) =$

$$\frac{1}{3}\cdot\frac{4}{10} + \frac{1}{3}\cdot\frac{3}{5} + \frac{1}{3}\cdot\frac{2}{6}$$

$$= \frac{4}{9}$$

b) $P(B \mid \underline{W})$

$$= \frac{\frac{1}{3}\cdot\frac{3}{5}}{\frac{4}{9}}$$

$$= \frac{9}{20}$$

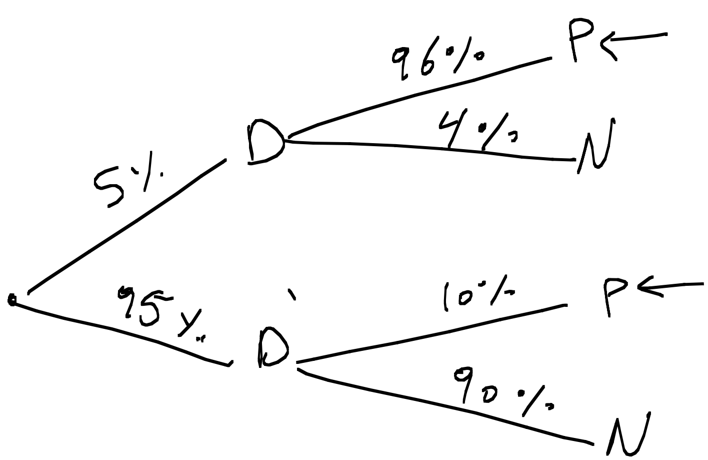
Example 9: An airline company is planning to screen all employees for the use of illegal drugs. The test has two results positive or negative. Positive result indicates that illegal drugs were used while negative result indicates no illegal drugs were used. The lab that is doing the test found from previous record that:

If a person did use illegal drugs, the test will detect is (show positive result) in 96% of the cases
(*this means: a* ***false negative*** *of 4% because it shows negative in 4% even though the person is a drug user)*

If a person did not use illegal drugs, the test will still show positive result in 10% of the cases
(this means a ***false positive*** *of 10%)*

At the end of the test, it was found that 5% of the employee did **actually** used illegal drugs. What is the probability:

a) that a person who tests positive actually did use illegal drugs?
b) that a person who actually use illegal drugs tests negative ?
c) that a person who ~~tests n~~egative actually did not use illegal drugs?
d) that a person who actually did not use illegal drugs tests positive ?



a) $P(D \mid P) = \dfrac{(5)(96)}{(5)(96) + (95)(10)}$

b) $P(N \mid D) = 4\%$

c) $P(D' \mid N) = \dfrac{(95)(90)}{(5)(4) + (95)(90)}$

d) $P(P \mid D') = 10\%$