Chapter 4

Section 4.1: Probability

Outcomes : a particular result of an experience**.**

Sample Space: the set of all possible outcomes of an experiment.

Example 1: * By rolling a die once, the sample space is $S = (1,2,3,4,5,6)$

* By flipping a coin twice, the sample space is $S =$ (HH,HT,TH,TT)

The Probability or (*likelihood*) of the occurrence of an event is:

$$\boxed{\text{Probability of an event } P(E) = \frac{n(E)}{n(S)}}$$

n(E): number of outcomes where the event occurs.
n(S): total number of possible outcomes in the sample space.

Example 2: In a survey of 100 people, it was found that 57 watch the late news.

$P(E)$: probability that a person watches late news $P(E) = 57\%$
$P(E')$: probability that a person does not watch late news $P(E') = 43\%$

$$P(E) + P(E') = 1$$

Equally Likely: When each of the outcomes of an experiment has the same probability of occurring (*fair die*, *fair coin*....)

Hint: In chapter 3, we were finding: (how many different ways) using permutation or combination. We will use the same concept in probability, but by dividing two numbers:

$$\text{Probability} = \frac{\text{Number of choices of what we are looking for}}{\text{Number of All possible choices}}$$

$$= \frac{\text{Number of All choices (with restriction)}}{\text{Number of All choices (with no restrictions)}}$$

Example 3: A team of 5 people to be selected out of 4 women and 7 men.

a) In **how many different ways** this can be done if there is no restrictions?

b) In **how many different ways** this can be done if the team must have 2 women?

c) What is the **probability** that the team has 2 women?

Example 4: By rolling a pair of dice, in how many different ways the sum is 4, 6, 9, or 12?

<u>Example 5</u>: By rolling a pair of dice, find all outcomes of sums and the probability of (

Sum of	By	# of ways	Prob.
2	11	1	1/36
3	21, 12	2	2/36
4	22, 31, 13	3	3/36
5	32, 23, 41, 14	4	4/36
6	33, 42, 24, 51, 15	5	5/36
7	43, 34, 61,16, 52, 25	6	6/36
8	44, 53, 35, 62, 26	5	5/36
9	54, 45, 63, 36	4	4/36
10	55, 64, 46	3	3/36
11	56, 65	2	2/36
12	66	1	1/36
	Sum =	36	36/36 = 1

Total number of events is 36 or 6x6 = 36

The sum of probabilities for <u>all</u> outcomes is always = 1

Example 6: By rolling a pair of dice, find the probability of:

a) getting the sum of 6.

b) not getting the sum of 6

Example 7: By selecting 5 cards, find the probability of getting:

a) exactly 3 Aces

b) same color

c) same suit

<u>Example 8</u>: In a box there are 15 Science books and 10 History. If 7 books are selected at random (*equally likely*), find the probability of getting at least 1 Science book.

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$P(A \cup B)$ = probability of A **or** B (either A or B or both)

$P(A \cap B)$ = probability of both A **and** B
= $P(A).P(B)$; if they are independent *(will be in section 4.2)*
= 0 ; if they are **mutually exclusive** (*disjoint*)

$P(A \cup B)'$ = probability of neither A nor B = $1 - P(A \cup B)$

Example 9: Out of 90 students surveyed, 30 took Math, 40 took English and 10 took both. What is the probability that a student took:

a) English and Math

b) neither English nor Math

Example 10: The probability that Bob will pass the Math course is 0.6, and that he will pass the English course is 0.7. If the probability that he will pass both of them is 0.4, find the probability that:

a) he will pass at least one course.

b) he will not pass any of the courses

c) he will pass either course but not both (only one)

<u>Example 11:</u> A survey in a college found that 40% passed the Math test , 70% passed the English test and 10% passed neither test. What is the probability :

a) of students that passed both test?

b) of students that passed one subject only?

Example 12: Using the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11. If one number is selected, what is the probability that it is less than 4 **or** odd?

The Odds:

- *Given the probability, find the odds*: If the probability of an event E is p, then

Odds for the event $= \dfrac{p}{1-p}$	Odds against the event $= \dfrac{1-p}{p}$

Example 13: The probability for winning a game is P(W) = 7/12. What is the odds:

a) for winning

b) against winning

- *Given the odds, find the probability*: If the odds for making an event E are a to b, then:

Probability of $(E) = \dfrac{a}{a+b}$	Probability of $(E') = \dfrac{b}{a+b}$

Example 14: The odds for winning a game is 7/5. What is the probability of:

a) winning

b) loosing