

Chapter 1 Introduction to Logic

Section 1.1

Statement: is a declarative sentence, which has a truth value; that is, it is either true or false, but not both true and false.

The following symbols will be used in logic: p , q , r , and sometimes s where:

$p \vee q$: read it as p or q or both (Disjunction)

$p \wedge q$: read it as p and q (Conjunction)

$\sim p$: read it as not p (Negation)

Example 1: p = Mike likes coffee ; q = Mike likes tea

- a) Mike likes coffee and he likes tea too.

- b) Mike likes coffee and he does not like tea

- c) Mike does not like coffee and he does not like tea.

There will be 4 rules regarding Chapter 1:

Rules 1 & 2: \vee *or* ; \wedge *And* (section 1.1)

Rules 3 & 4: \rightarrow *If* ; \leftrightarrow *If and only if* (section 1.2)

Rule 1: Using the \vee symbol (**or**), it is true when either one or both are true.

$$\begin{aligned}T \vee T &= T \\T \vee F &= T \\F \vee T &= T \\F \vee F &= F\end{aligned}$$

Rule 2: Using the \wedge symbol (**and**), it is true only when both are true.

$$\begin{aligned}T \wedge T &= T \\T \wedge F &= F \\F \wedge T &= F \\F \wedge F &= F\end{aligned}$$

In creating the truth table:

The number of rows in the truth table depends on the number of variables n :

For p and q ($n = 2$ variables), number of rows = 4

For p, q and r ($n = 3$ variables), number of rows = 8

For p, q, r and s ($n = 4$ variables), number of rows = 16

The above numbers were found using the formula of 2^n which will be used in chapter 2 in finding the number of subsets.

For p and q with 4 rows: The first column 2 T and 2 F, the second column 1 T and 1 F.

p	q
T	T
T	F
F	T
F	F

For p, q and r with 8 rows: The first column 4 T and 4 F, the second column 2 T and 2 F and the third column 1 T and 1 F

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

- Example 2: Construct the truth table for:

a) $\sim(p \wedge \sim q) \vee p$

p	q				
T	T				
T	F				
F	T				
F	F				

b) $(p \vee q) \wedge (p \vee \sim r)$

p	q	r				
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Tautology = valid argument: is a statement that is true for all possible combinations of truth conditions for the component statement (*the elements of the last column are all T*). See example 2a.

Contradiction: is a statement that is false for all possible combinations of truth conditions for the component statement (*The elements of the last column are all F*).

Logical Equivalence: When they have identical truth values under identical truth conditions of the simple statement (*When two statements have identical last column in the truth tables*). See example 3.

- **Example 3: Construct the truth table for:**

a) $\sim(p \wedge q)$

<i>p</i>	<i>q</i>		
T	T		
T	F		
F	T		
F	F		

b) $\sim p \vee \sim q$

<i>p</i>	<i>q</i>		
T	T		
T	F		
F	T		
F	F		