Chapter 5

Section 5.1: Central Tendency

Mode: the number or numbers that occur **most often**.

Median: the number at the **midpoint** of a ranked data.

Example 1: The test scores for a test were: 78, 81, 82, 76, 84, 81, 76. Find the mode and the median.

The **mode** is 81 and 76, both of them repeated twice

The **median** must be found after the data is ranked from smallest to largest.
For the above data: 76, 76, 78, 81, 81, 82, 84 the median is 81 which is located in the middle.

Example 2: The test scores for a test were: 78, 81, 82, 76, 84, 86. Find the mode and the median.

There is no **mode**, no score is repeated more than once

The **median** must be found after the data is ranked from smallest to largest.
For the above data: 76, 78, 81, 82, 84, 86. There are two values in the middle 81 and 82,
then the median is average of those two values or (81+82)/2= 81.5

Example 3: The test scores for a test were: 78, 78, 78, 81, 81, 95, 95, 95, 100. Find the total.
As you noticed, there are repeated scores and it is easier to find the total of those scores this way:

Total = 3(78) + 2(81) + 3(95) + 1(100) = 781

Or: total = $\Sigma f_i x_i$

where: Σ is the symbol for sum

x_i is the score; f_i is the frequency of each score

Example 4: Find the average score for the tests in example 3.

The average score is the total divided by the number of tests, there are 9 tests,

The average is = (781)/9 = 86.78

Or: $$\bar{x} = \frac{\sum f_i x_i}{n}$$

where n is number of tests, $n = \Sigma f_i$ *(sum of frequencies)*

<u>Section 5.2</u> Expected Value and Standard Deviation

Random Variable: A function *X* that assigns to every outcome exactly one real number.

Probability Density Function: A list of all possible values of the random variables and the associated probabilities.

Outcomes (events)	Random Variable (X)	Prob. Density (P)
all possibilities	value of each possibility	prob. of each possibility
		Sum = 1

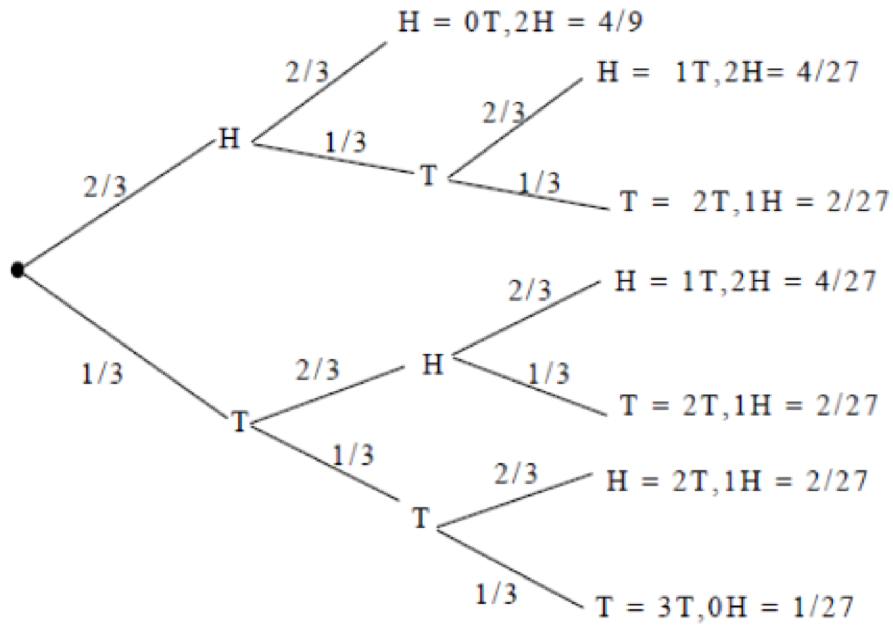
<u>Example 1:</u> An unfair coin in which P(*H*) = 2/3 is flipped twice. The random variable *X* is defined to be the number of heads. Find the density function.

X = the number of heads

Outcomes	X	P
HH	2	$2/3 \cdot 2/3 = 4/9$
HT or TH	1	$2/3 \cdot 1/3 + 1/3 \cdot 2/3 = 4/9$
TT	0	$1/3 \cdot 1/3 = 1/9$

Using the Tree*: Use it when the problem is written is way that the experience stops when certain condition is met. See the next example and how the word "until" is an indication of tree is needed.*

Example 2: An experiment consists of flipping an unfair coin where P(*H*) = 2/3 **until** a total of 2 heads occur or 3 flips. The random variable is defined to be the number of tails. Find the expected value of the random variable



Outcomes	*X*	P	*X*P
0T, 2 H	0	4/9	0
1T, 2 H	1	4/27 + 4/27 = 8/27	8/27
2T, 1 H	2	2/27 + 2/27 + 2/27 = 6/27	12/27
3T, 0 H	3	1/27	3/27
	Sum	= 1	E(*X*) = 23/27 = 0.85

Binomial & Non-Binomial Distribution *(Using Tables)*

- **Expected Value E[X], μ:** The mean, the average value of a random variable in which:
  $\mu = E[X] = \sum P_i X_i = X_1P_1 + X_2P_2 + X_3P_3 + \ ....$
- **Variance** σ^2 **:** $\sigma^2 = \sum P_i(X_i - \mu)^2$ **The Variance is a measure of the dispersion of the distribution of a random variable.**
- **Standard Deviation:** $\sigma = \sqrt{\sigma^2}$

Probability Types (from chapter 4):

1) Binomial Distribution: The probability of section 4.4 or Bernoulli trials:

(repeated events) is applied and the probability of the repeated events **is the same**:

Common example: flipping coins, or when applying same given probability on all selected parts. See example 3

2) Non Binomial Distribution: The probability of section 4.1:

$$\text{Probability} = \frac{\text{Number of choices of what we are looking for}}{\text{Number of All possible choices}}$$

Common example: Selecting team of people, cards when the probabilty changes (first card is out of 52, second is out of 51 and so on) See example 5.

Example 3: Stereo speakers manufactured with probability of 20% being defective. Three are selected off continuous assembly line, define the random variable X as the number of the defective parts. Find:

a) the density function and the expected value for the defective parts

b) the expected value for the good parts

c) the variance, the standard deviation.

a) X = *number of defective parts*

Outcomes	X	P	$X.P$
0D, 3G	0	$C(3,0).(0.80)^3.(0.20)^0 = 0.51$	0
1D, 2G	1	$C(3,1).(0.80)^2.(0.20)^1 = 0.38$	0.38
2D, 1G	2	$C(3,2).(0.80)^1.(0.20)^2 = 0.096$	0.192
3D, 0G	3	$C(3,3).(0.80)^0.(0.20)^3 = 0.008$	0.024
	Sum	= 1	= 0.6

Expected value for the defective part is = 0.6

b) Expected value for the good part is = 3 - 0.6 = 2.4

Example 3Cont.:

X	P
0	$C(3,0).(0.80)^3.(0.20)^0 = 0.51$
1	$C(3,1).(0.80)^2.(0.20)^1 = 0.38$
2	$C(3,2).(0.80)^1.(0.20)^2 = 0.096$
3	$C(3,3).(0.80)^0.(0.20)^3 = 0.008$

c)

X	P	$(X_i - \mu)$	$(X_i - \mu)^2$	$P_i(X_i - \mu)^2$
0	0.51	(0 -0.6) = -0.6	$(-0.6)^2 = 0.36$	0.51(0.36)= 0.184
1	0.38	(1 -0.6) = 0.4	$(0.4)^2 = 0.16$	0.38 (0.16)= 0.061
2	0.096	(2 -0.6) = 1.4	$(1.4)^2 = 1.96$	0.096 (1.96)= 0.19
3	0.008	(3 -0.6) = 2.4	$(2.4)^2 = 5.76$	0.008 (5.76)= 0.05
			Sum =	0.48

Variance σ^2: $\sigma^2 = \sum P_i(X_i - \mu)^2 = 0.48$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.48} = 0.69$

Binomial Distribution ***(Using Formula)***

- **Expected Value** $E[X]$ or μ where : $E[X] = \mu = n \cdot p$
- **Variance** σ^2 where $\sigma^2 = n \cdot p \cdot q$
- **Standard Deviation:** $\sigma = \sqrt{\sigma^2}$

Example 4: Solve example 3 again but without table

This problem is Binomial, first find *n,p & q:* $n = 3$, $p = 0.2$, $q = 0.8$

Expected Value: $\mu = n \cdot p = 3(0.2) = 0.6$ for the defective parts

Variance: $\sigma^2 = n \cdot p \cdot q = 3(0.2)(0.8) = 0.48$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.48} = 0.69$

Example 5: A box with 6 good parts and 4 defective in which 3 are selected. The random variable X is defined as the number of defective parts selected. Find:

a) the density function and the expected value for the defective parts
b) the expected value for the good parts
c) the variance, the standard deviation.

This problem is **not Binomial**.

X = number of defective parts

Outcomes	X	P	$X.P$
0D,3G	0	$\frac{C(4,0).C(6,3)}{C(10,3)} = \frac{20}{120}$	0
1D,2G	1	$\frac{C(4,1).C(6,2)}{C(10,3)} = \frac{60}{120}$	$\frac{60}{120}$
2D,1G	2	$\frac{C(4,2).C(6,1)}{C(10,3)} = \frac{36}{120}$	$\frac{72}{120}$
3D,0G	3	$\frac{C(4,3).C(6,0)}{C(10,3)} = \frac{4}{120}$	$\frac{12}{120}$
	Sum	= 1	= 1.2

Expected value for the defective part is = 1.2

b) Expected value for the good part is = 3 - 1.2 = 1.8

Example 5 Cont.:

X	P
0	$\frac{C(4,0).C(6,3)}{C(10,3)} = \frac{20}{120}$
1	$\frac{C(4,1).C(6,2)}{C(10,3)} = \frac{60}{120}$
2	$\frac{C(4,2).C(6,1)}{C(10,3)} = \frac{36}{120}$
3	$\frac{C(4,3).C(6,0)}{C(10,3)} = \frac{4}{120}$

c)

X	P	$(X_i - \mu)$	$(X_i - \mu)^2$	$P_i(X_i - \mu)^2$
0	$\frac{20}{120}$	(0 -1.2) = -1.2	$(-1.2)^2=1.44$	$\frac{20}{120}(1.44)= 0.24$
1	$\frac{60}{120}$	(1 -1.2) = -0.2	$(-0.2)^2=0.44$	$\frac{60}{120}(0.44)= 0.02$
2	$\frac{36}{120}$	(2 -1.2) = 0.8	$(0.8)^2=0.64$	$\frac{36}{120}(0.64)= 0.192$
3	$\frac{4}{120}$	(3 -1.2) = 1.8	$(1.8)^2=3.24$	$\frac{4}{120}(3.24)= 0.108$
			Sum =	0.56

Variance σ^2: $\sigma^2 = \sum P_i(X_i - \mu)^2 = 0.56$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.56} = 0.748$

Example 6: A multiple-choice test contains 10 questions with 4 choices for each answer. If a student guesses the answers, find:

a) the probability that he will get 4 correct answers.

b) the expected value for the correct answers

c) the expected value for the wrong answers

d) the variance, the standard deviation.

This problem is a Binomial, find n,p & q: $n = 10$, $p = 1/4 = 0.25$, $q = 0.75$.

a) $P = C(10,4).(0.25)^4.(0.75)^6$

b) Expected Value: $\mu = n \cdot p = 10(0.25) = 2.5$ for the correct answers

c) Expected Value: $\mu = n \cdot p = 10(0.75) = 7.5$ for the wrong answers

d) Variance: $\sigma^2 = n \cdot p.q = 10(0.25)(0.75) = 1.875$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{1.875} = 1.37$

Example 7: Two coins are selected at random from a pocket that contains 2 nickels and 6 quarters. The random variable *X* is the total value in cents of the 2 selected coins. Find E(*X*).

Example 8: By rolling a pair of dice, a game is played in which:

You win $2 if the sum is 2, 3, 4 or 5.

You win $3 if the sum is 6, 7 or 8.

You loose $5 if the sum is 9, 10, 11 or 12.

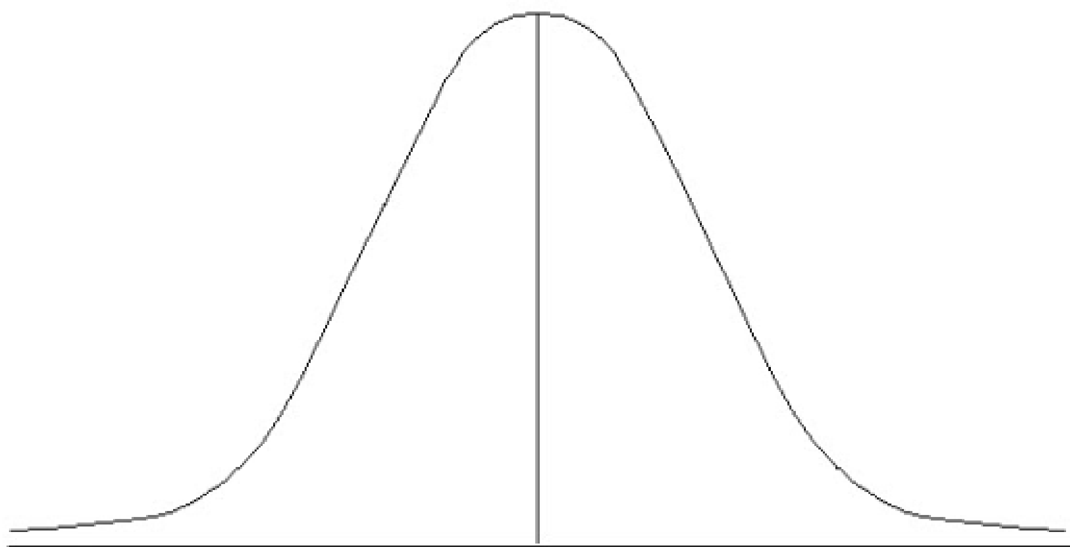
If you pay $2 to play the game, find the expect gain or loss.

Outcomes	*X*	P	*X*.P
2, 3, 4, 5	+2	1/36 + 2/36 + 3/36 + 4/36 = 10/36	20/36
6, 7, 8	+3	5/36 + 6/36 + 5/36 = 16/36	48/36
9, 10, 11, 12	-5	4/36 + 3/36 + 2/36 + 1/35 = 10/36	-50/36
	Sum	1	E(*X*)=$ 0.5

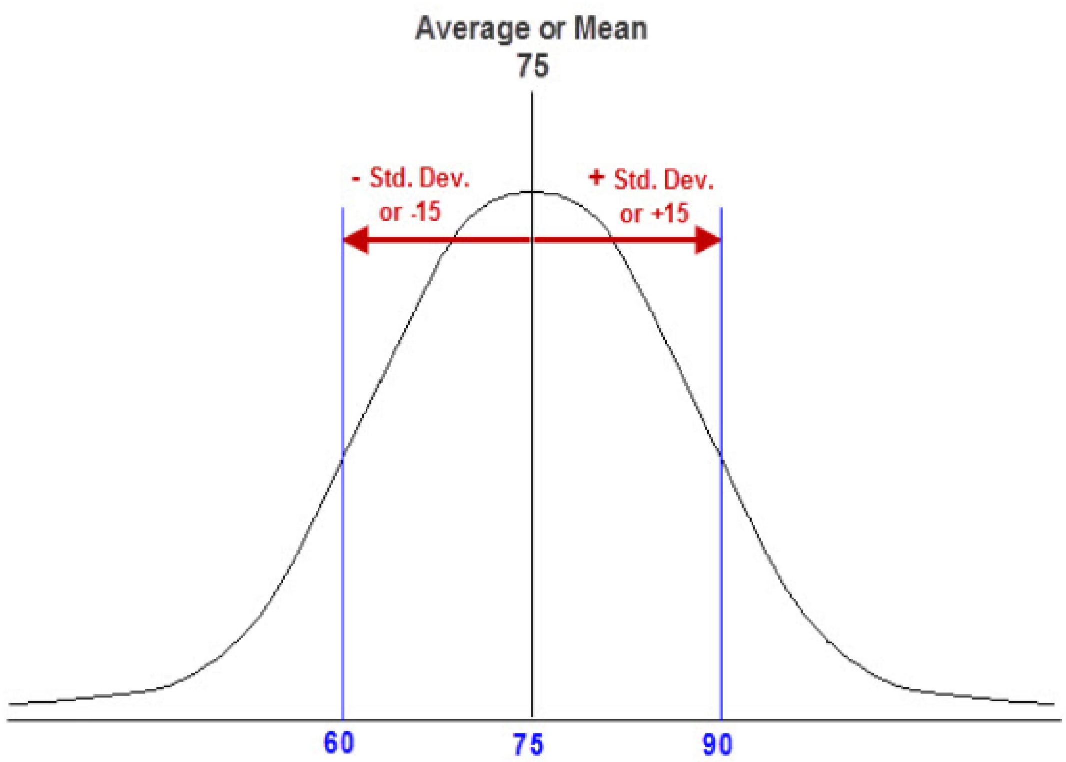
Expected gain is $0.5, but you paid $2 to play the game, then there is a loss of $1.5

<u>Section 5.3:</u> Normal Random Variable

<u>Example:</u> If the test average is 75, and the standard devistion is 15 and the scores are normally distributed (Bell Shaped curve)

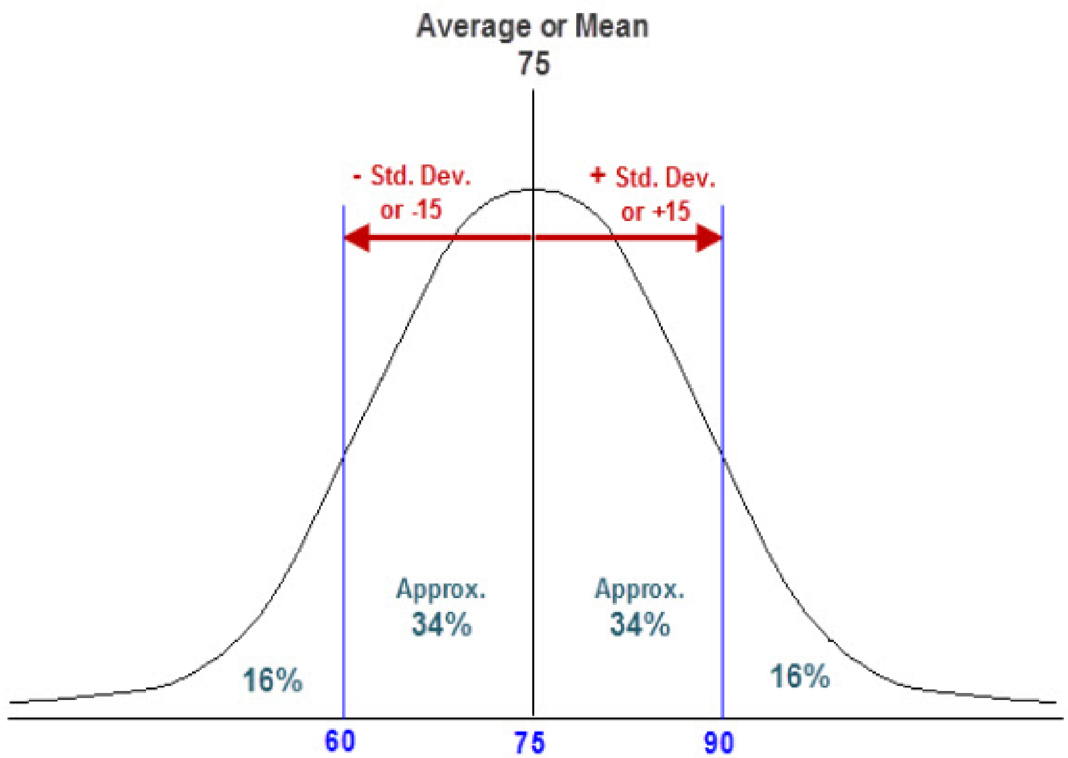


Example: If the test average is 75, and the standard devistion is 15 and the scores are normally distributed (Bell Shaped curve)



Standard deviation and confidence intervals

About 68% of values drawn from a normal distribution are within one standard deviation away from the mean μ;



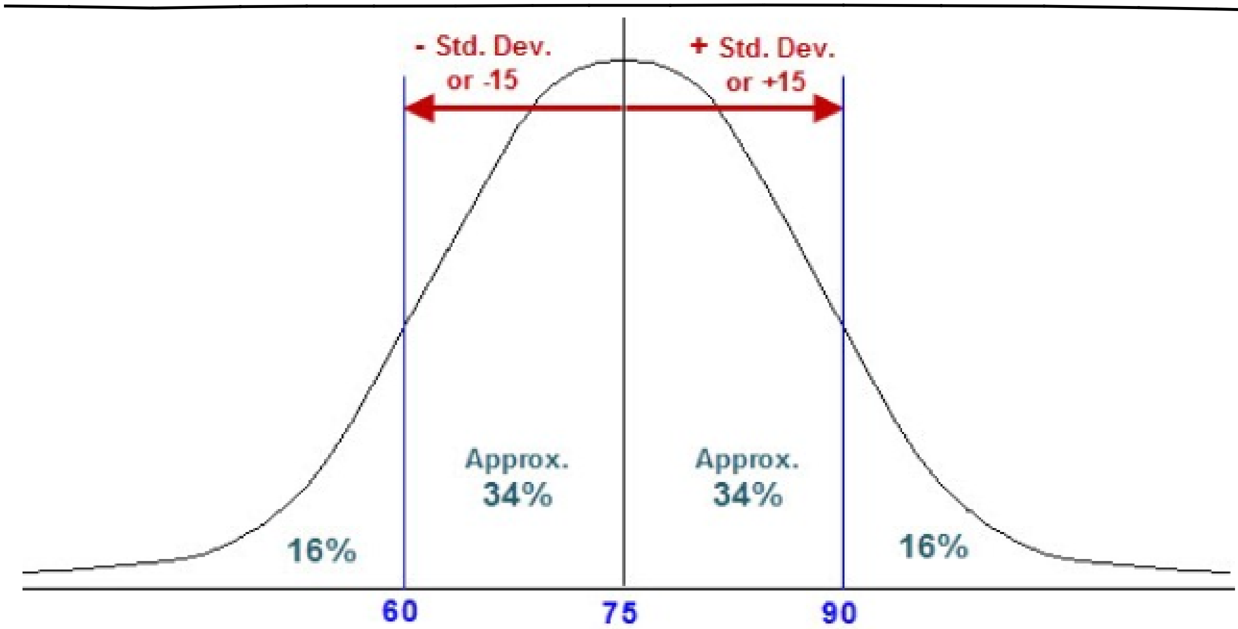
$$Z = \frac{X - \mu}{\sigma}$$

<u>Example:</u> If the test average is 75, and the standard devistion is 15, find the Z-scores for the following scores:

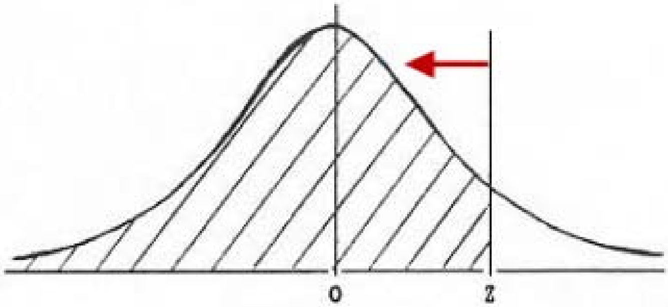
a) x = 90

b) x = 60

c) x = 75



Areas under the Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Area Under The Standard Normal Curve ($Z \geq 0$)

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993

Area Under The Standard Normal Curve ($Z \leq 0$)

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Example 1: Let Z be a random variable with normal distribution. Using the table, find:

a) $P(Z \leq 1.87)$

b) $P(0.49 \leq Z \leq 1.75)$

c) $P(-1.77 \leq Z \leq 2.53)$

Example 1 (Cont.): Let Z be a random variable with normal distribution. Using the table, find:

d) $P(Z \geq 1.87)$

e) $P(-1.00 \leq Z \leq 1.00)$

f) $P(0.00 \leq Z \leq 2.17)$

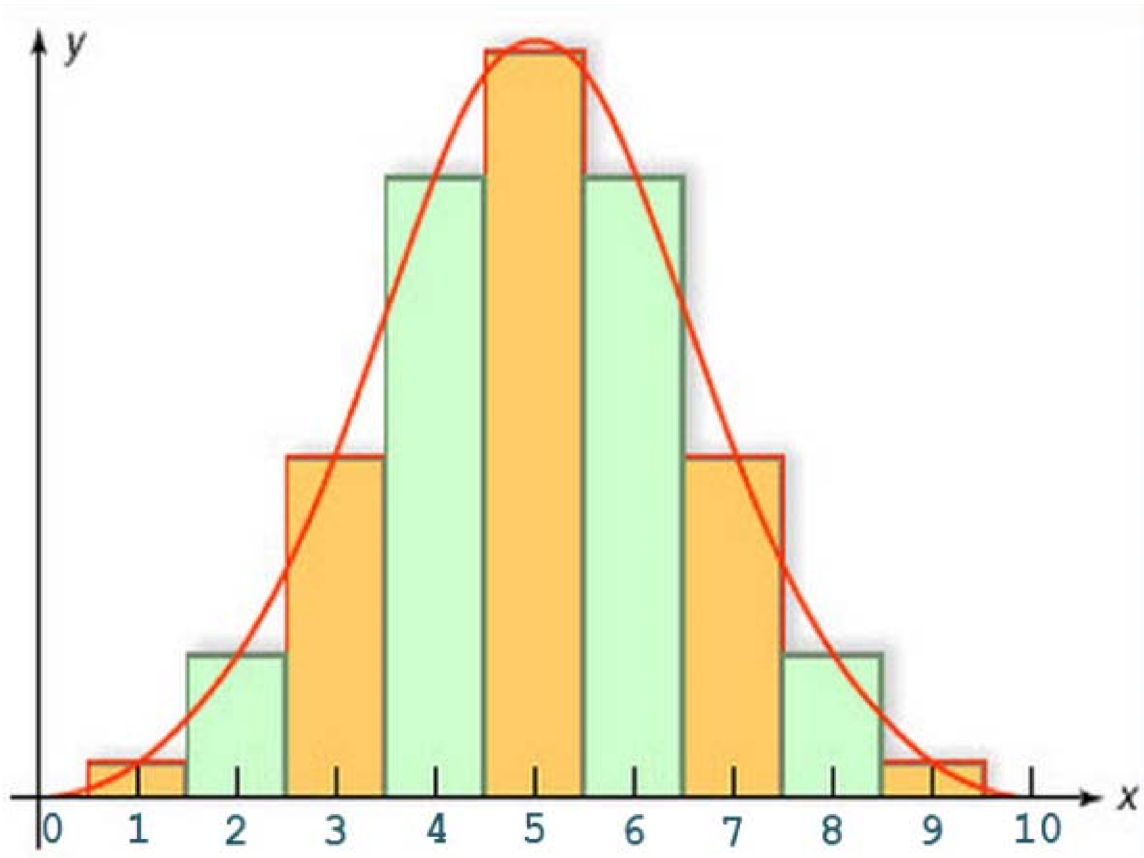
g) $P(Z \geq 2.17)$

Example 2: Suppose that for a certain population the birth weight of infants in pounds is normally distributed with mean 7.75 pounds and standard deviation of 1.25 pounds. Find the probability that an infant's birth is at least 9

Example 3: Bolts produced by a machine are acceptable provided that their length is within the range 5.95 to 6.05 inches. Suppose that the length of the bolts produced are normally distributed with mean of 6 inches and standard deviation of 0.02 inches. what is the probability that a bolt will be an acceptable length?

Example 4: Let be a normal random variable with mean 400 and standard deviation 40. Find Pr[$360 \leq X \leq 410$]. Give answer as a decimal number correct to three decimal places

<u>Section 5.4:</u> Normal Approximation To The Binomial



<u>RULES:</u> To approximate **<u>binomial</u>** probability by normal curve area:

Step 1) determine n, p, q

Step 2) check that both $n.p > 5$ <u>and</u> $n.q > 5$

Step 3) find the expected value and the standard deviation

$$\mu = n \cdot p \qquad\qquad \sigma = \sqrt{n.p.q}$$

Step 4) find the new points by:

* subtracting 0.5 from the starting point

* adding 0.5 to the finish point

examples: $P(3 \le X \le 6)$ *will be* $P(2.5 \le X \le 6.5)$

$P(X = 7)$ *will be* $P(6.5 \le X \le 7.5)$

$P(X \ge 8)$ *will be* $P(X \ge 7.5)$

$P(X \le 8)$ *will be* $P(X \le 8.5)$

Step 5) find the Z-scores and the area under the normal curve using the table

Example 1: According to the Department of Health and Human Services, the probability is about 80% that a person aged 70 will be alive at the age of 75. Suppose that 500 people aged 70 are selected at random. Find the probability that:

a) exactly 390 of them will be alive at the age of 75

a) Step 1) $n = 500$, $p = 0.8$, $q = 0.2$

Step 2) check if both $n.p$ and $n.q$ are more than 5:

$$n.p = (500).(0.8) = 400$$

$$n.q = (500).(0.2) = 100$$

Step 3) find the expected value and the std. deviation:

$$\mu = n \cdot p = (500).(0.8) = 400$$

$$\sigma = \sqrt{n.p.q} = \sqrt{(500).(0.8).(0.2)} = 8.94$$

Step 4) find the new point:

$P(X = 390)$ will be $P(389.5 \le X \le 390.5)$

Step 5) find the Z-score:

$X = 389.5$, $Z = \dfrac{389.5 - 400}{8.94} = -1.17$

$X = 390.5$, $Z = \dfrac{390.5 - 400}{8.94} = -1.06$

and now by using the table:

$P(-1.17 \le Z \le -1.06) = 0.1446 - 0.1210 = 0.0236$

Example 1 (Cont.) : According to the Department of Health and Human Services, the probability is about 80% that a person aged 70 will be alive at the age of 75. Suppose that 500 people aged 70 are selected at random. Find the probability that:

b) for $P(375 \leq X \leq 425)$, we use the information of steps 1, 2 and 3 then:

$P(375 \leq X \leq 425)$ will be $P(374.5 \leq X \leq 425.5)$

$$X = 374.5, \qquad Z = \frac{374.5 - 400}{8.94} = -2.85$$

$$X = 425.5, \qquad Z = \frac{425.5 - 400}{8.94} = 2.85$$

and now by using the table:

$P(-2.85 \leq Z \leq 2.85) = 0.9978 - .0022 = 0.9956$

	Section 5.3 No Approximation	**Section 5.4** Approximation
Given	Expected value Standard Deviation	n, p
Steps	• Find: Z-Score: $Z = \frac{X - \mu}{\sigma}$ • Use the table	• Find: $q \text{ where } q = 1 - p$ excepted value $E[X] = \mu = n \cdot p$ Standard deviation $\sigma = \sqrt{n.p.q}$ • Add / subtract 0.5 as needed • Find the Z-Score : $Z = \frac{X - \mu}{\sigma}$ • Use the table

Example 2: A coin with Pr[Tails] = 0.4 is flipped 200 times. Find the probability of getting between 65 and 100 tails on the coin. Give your answer as a decimal number correct to three decimal places

Example 3: Assume that IQ scores are normally distributed with mean 100 and standard deviation 15. What is the probability that a randomly chosen person will have an IQ at most 105?