

## Chapter 1 Introduction to Logic

### Section 1.1

**Statement**: is a declarative sentence, which has a truth value; that is, it is either true or false, but not both true and false.

The following symbols will be used in logic:  $p$ ,  $q$ ,  $r$ , and sometimes  $s$  where:

$p \vee q$  : read it as  $p$  or  $q$  or both (Disjunction)

$p \wedge q$  : read it as  $p$  and  $q$  (Conjunction)

$\sim p$  : read it as not  $p$  (Negation)

**Example 1:**  $p$  = Mike likes coffee ;  $q$  = Mike likes tea

- a) Mike likes coffee and he likes tea too.
  
- b) Mike likes coffee and he does not like tea
  
- c) Mike does not like coffee and he does not like tea.

**There will be 4 rules regarding Chapter 1:**

**Rules 1 & 2:**  $\vee$  *or* ;  $\wedge$  *And* (section 1.1)

**Rules 3 & 4:**  $\rightarrow$  *If* ;  $\leftrightarrow$  *If and only if* (section 1.2)

**Rule 1:** Using the  $\vee$  symbol (**or**), it is true when either one or both are true.

$$\begin{aligned}T \vee T &= T \\T \vee F &= T \\F \vee T &= T \\F \vee F &= F\end{aligned}$$

**Rule 2:** Using the  $\wedge$  symbol (**and**), it is true only when both are true.

$$\begin{aligned}T \wedge T &= T \\T \wedge F &= F \\F \wedge T &= F \\F \wedge F &= F\end{aligned}$$

**In creating the truth table:**

**The number of rows in the truth table depends on the number of variables  $n$ :**

**For  $p$  and  $q$  ( $n = 2$  variables), number of rows = 4**

**For  $p, q$  and  $r$  ( $n = 3$  variables), number of rows = 8**

**For  $p, q, r$  and  $s$  ( $n = 4$  variables), number of rows = 16**

**The above numbers were found using the formula of  $2^n$  which will be used in chapter 2 in finding the number of subsets.**

**For  $p$  and  $q$  with 4 rows: The first column 2 T and 2 F, the second column 1 T and 1 F.**

$p$	$q$
T	T
T	F
F	T
F	F

**For  $p, q$  and  $r$  with 8 rows: The first column 4 T and 4 F, the second column 2 T and 2 F and the third column 1 T and 1 F**

$p$	$q$	$r$
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

- Example 2: Construct the truth table for:

a)  $\sim(p \wedge \sim q) \vee p$

$p$	$q$				
T	T				
T	F				
F	T				
F	F				

b)  $(p \vee q) \wedge (p \vee \sim r)$

$p$	$q$	$r$				
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

**Tautology = valid argument:** is a statement that is true for all possible combinations of truth conditions for the component statement (*the elements of the last column are all T*). See example 2a.

**Contradiction:** is a statement that is false for all possible combinations of truth conditions for the component statement (*The elements of the last column are all F*).

**Logical Equivalence:** When they have identical truth values under identical truth conditions of the simple statement (*When two statements have identical last column in the truth tables*). See example 3.

- **Example 3: Construct the truth table for:**

a)  $\sim(p \wedge q)$

<i>p</i>	<i>q</i>		
T	T		
T	F		
F	T		
F	F		

b)  $\sim p \vee \sim q$

<i>p</i>	<i>q</i>		
T	T		
T	F		
F	T		
F	F		

## Section 1.2:

Example 1:

$p = \text{T}$       Mike was caught cheating in the test

$q = \text{T}$       Mike was suspended from school

Two variables, 4 possibilities:

- a) If Mike was caught cheating, then he will be suspended from school.
- b) If Mike was caught cheating, then he will not suspended from school
- c) If Mike was not caught cheating, then he will be suspended from school
- d) If Mike was not caught cheating, then he will not be suspended from school

	$p$	$q$	if $p$ then $q$ $p \rightarrow q$
a)	T	T	T
b)	T	F	F
c)	F	T	T
d)	F	F	T

- Rule 3: Using the  $\rightarrow$  symbol ( If ), it is always true unless the first is T and the second is F.

**Example 2:** Construct the truth table of:  $(p \rightarrow q) \wedge (q \rightarrow p)$

	$p$	$q$	if $p$ then $q$ $p \rightarrow q$	if $q$ then $p$ $q \rightarrow p$	$(p \rightarrow q) \wedge$ $(q \rightarrow p)$
a)	T	T			
b)	T	F			
c)	F	T			
d)	F	F			

Result:

	$p$	$q$	if $p$ then $q$ $p \rightarrow q$	if $q$ then $p$ $q \rightarrow p$	$(p \rightarrow q) \wedge$ $(q \rightarrow p)$
a)	T	T	T	T	T
b)	T	F	F	T	F
c)	F	T	T	F	F
d)	F	F	T	T	T

The last column is the same as:  $(p \leftrightarrow q)$  which is (*if and only if*) which is **Biconditional**.

- **Rule 4:** Using the  $\leftrightarrow$  symbol (**if and only if**), it is true when both are T, or both are F.

**Example 3:** Translate this argument to symbolic form and construct the truth table and determine if it is a valid argument or not.

If it rains, then the concert will be delayed  
The concert was delayed, therefore it rained.

$p$	$q$			
T	T			
T	F			
F	T			
F	F			



**Example 4:** Translate this argument to symbolic form and construct the truth table and determine if it is a valid argument or not.

If you miss the final exam, then you will get a zero for the exam

If you get zero in the final exam, then you will not pass the course.

You did not miss the final exam, therefore you will pass

$p$	$q$	$r$							
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

**Example 5:** Translate this argument to symbolic form:

The alarm will sound if and only if smoke or carbon monoxide is in the house

There is no carbon monoxide in the house

Therefore, the alarm will sound if and only if smoke is in the house

Summary for Sections 1.1 and 1.2:

- **Logical Equivalence:** When they have identical truth values under identical truth conditions of the simple statement (*When two statements have identical last column in the truth tables*).
- **Tautology = valid argument:** is a statement that is true for all possible combinations of truth conditions for the component statement (*the elements of the last column are all T*)
- **Contradiction:** is a statement that is false for all possible combinations of truth conditions for the component statement (*The elements of the last column are all F*)
- **Conditional:**  $p \rightarrow q$  if  $p$  then  $q$
- **Biconditional:**  $p \leftrightarrow q$  If and only If  $p$  then  $q$

$p$ : The weather is cold       $q$ : You will wear a coat

- **Conditional:**  $p \rightarrow q$  if  $p$  then  $q$   
If the weather is cold, then you will wear a coat
- **Converse:**  $q \rightarrow p$   
If you wear a coat, then the weather is cold
- **Inverse:**  $\sim p \rightarrow \sim q$   
If the weather is not cold, then you will not wear a coat
- **Contrapositive:**  $\sim q \rightarrow \sim p$   
If you will not wear a coat, then the weather is not cold

				<i>Disjunction p or q</i>	<i>Conjunction p and q</i>	<i>Conditional if p then q</i>	<i>Converse if q then p</i>	<i>Inverse if <math>\sim q</math> then <math>\sim p</math></i>	<i>Contrapositive if <math>\sim p</math> then <math>\sim q</math></i>	<i>Biconditional if and only if</i>
$p$	$q$	$\sim p$	$\sim q$	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$	$p \leftrightarrow q$
T	T	F	F	T	T	T	T	T	T	T
T	F	F	T	T	F	F	T	T	F	F
F	T	T	F	T	F	T	F	F	T	F
F	F	T	T	F	F	T	T	T	T	T