Notes

Math M118: Lecture Notes For Chapter 7

Slope-Intercept Equation $y = mx + b$ *(m is the slope, b is the y-intercept)*

$m > 0$ or positive slope, then the line is increasing or rising, as $y = 2x - 3$

$m < 0$ or negative slope, then the line is decreasing or falling, as $y = -3x + 4$

$m = 0$, then the line is horizontal, as $y = 2$

$m =$ **undefined,** no slope, then the line is vertical, $x = 3$

b determines where the line crosses the y-axis: above ($b > 0$, *pos.*), below ($b < 0$, neg.) or through ($b = 0$).

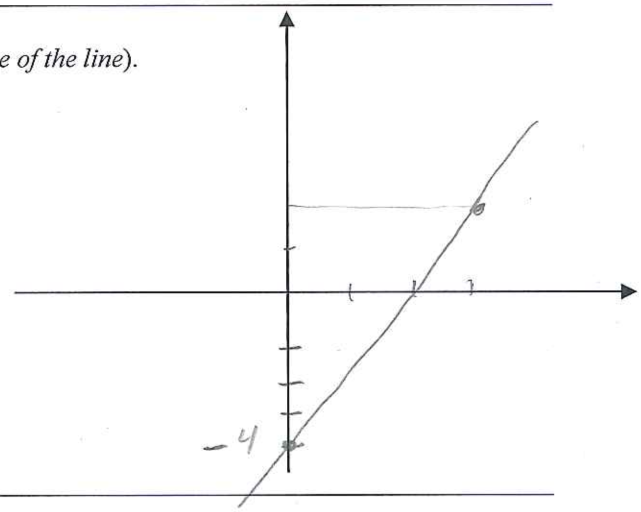
Example 1: Graph $2x - y = 4$, *(hint: isolate y first to know the shape of the line).*

$-y = -2x + 4$

$y = 2x - 4$

$m = 2$, $y_{int} = -4$

x	y
0	-4
3	2



Example 2: Match the following equations with the correct graph:

1) $y = -2x + 4$; graph: H

2) $y = 2x + 4$; graph: B

3) $y = -5x - 2$; graph: A

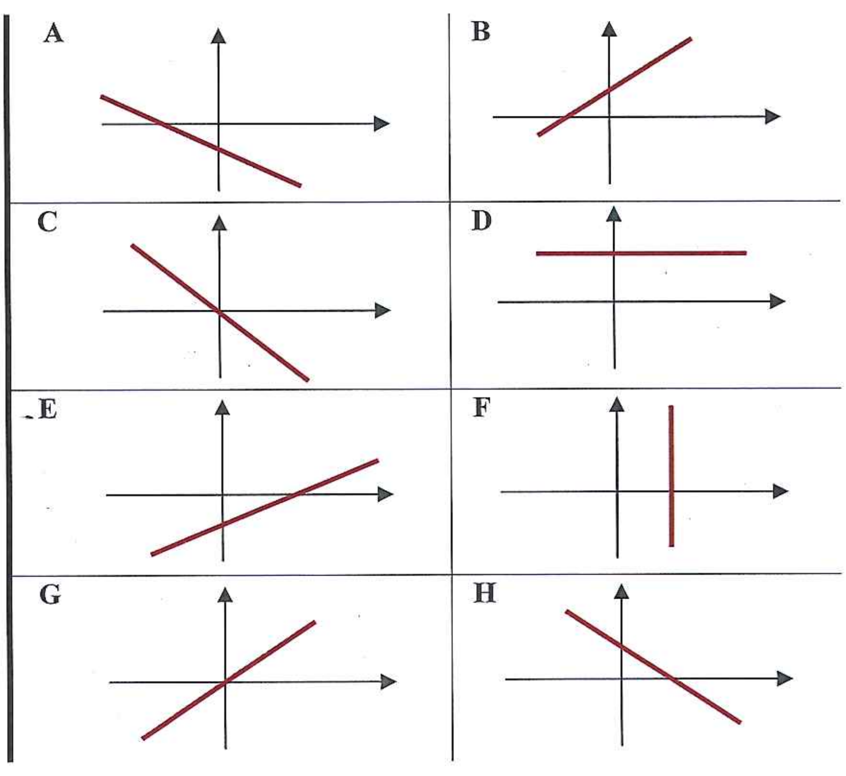
4) $y = 4x - 2$; graph: E

5) $y = -2x$; graph: C

6) $y = 3x$; graph: G

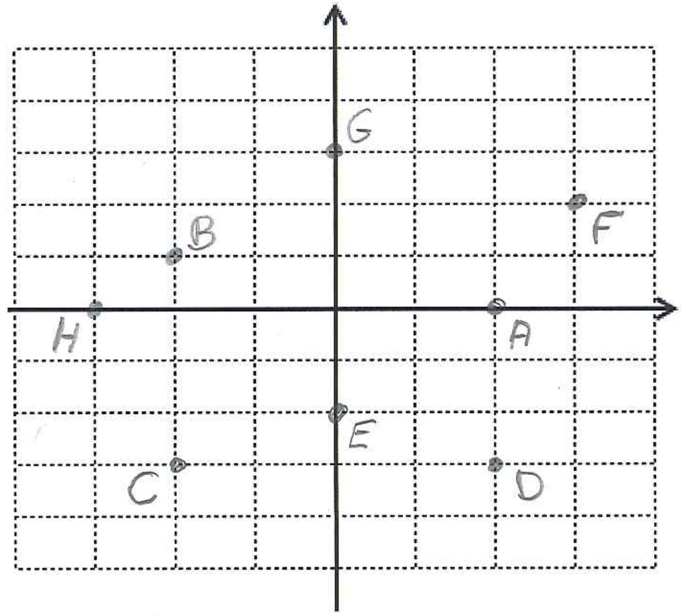
7) $y = 2$; graph: D

8) $x = 3$; graph: F



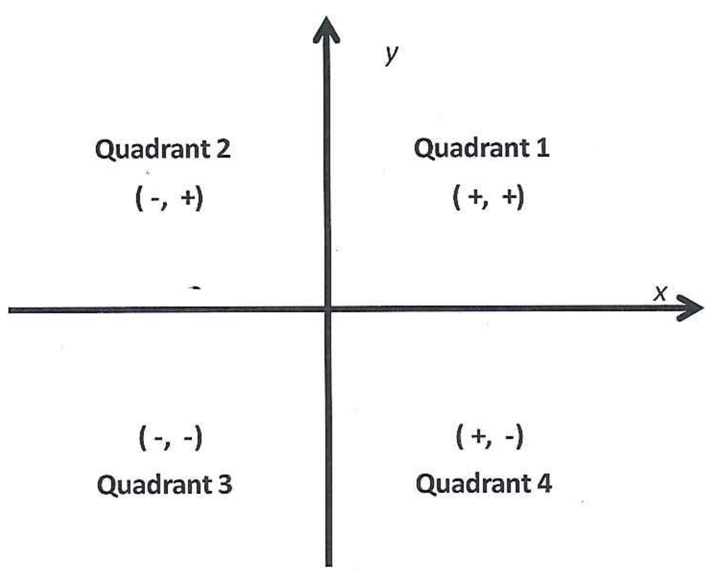
Example 3: Locate the following points:

1) A (2, 0)
2) B (-2, 1)
3) C (-2, -3)
4) D (2, -3)
5) E (0, -2)
6) F (3, 2)
7) G (0, 3)
8) H (-3, 0)



Notice:

- Any point on the y-axis has $x = 0$, or it is called the ***y*-intercept** — Points E and G
- Any point on the x-axis has $y = 0$, or it is called the ***x*-intercept** — Points A and H
- Points in the first quadrant has (+ , +), both positive x and y: — Point F, $x > 0$, $y > 0$
- Points in the second quadrant has (- , +), negative x, positive y: — Point B, $x < 0$, $y > 0$
- Points in the third quadrant has (- , -), both negatives x and y: — Point C, $x < 0$, $y < 0$
- Points in the fourth quadrant has (+ , -), positive x, negative y, — Point D, $x > 0$, $y < 0$



Important Notes

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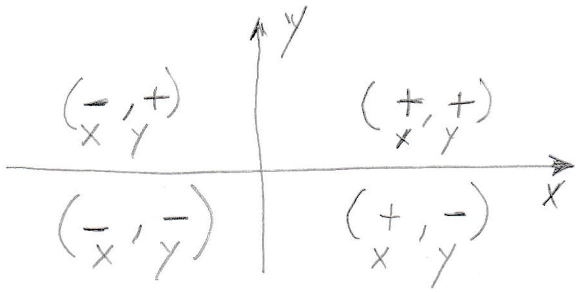

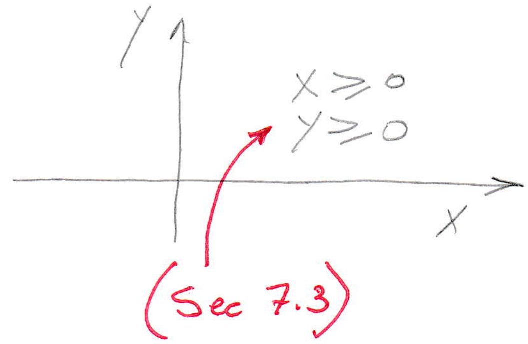

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* $m < 0$,

Any Point on $y \longrightarrow x = 0$

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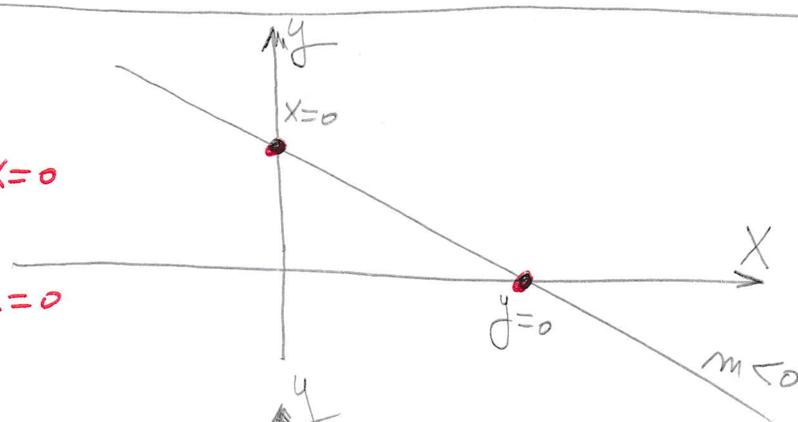
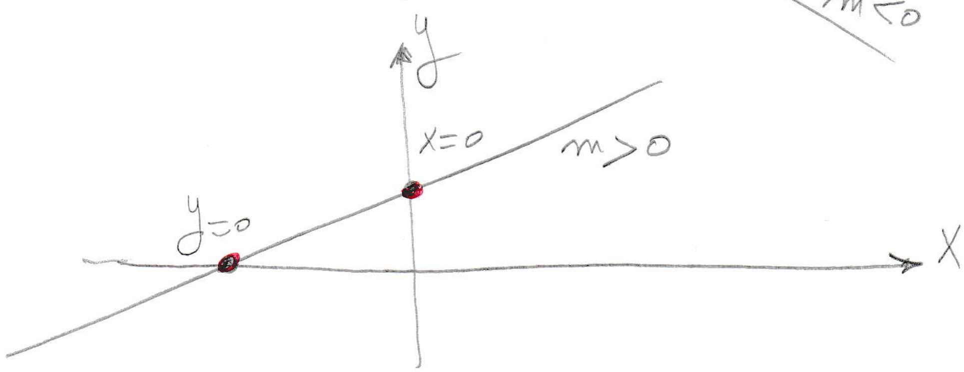
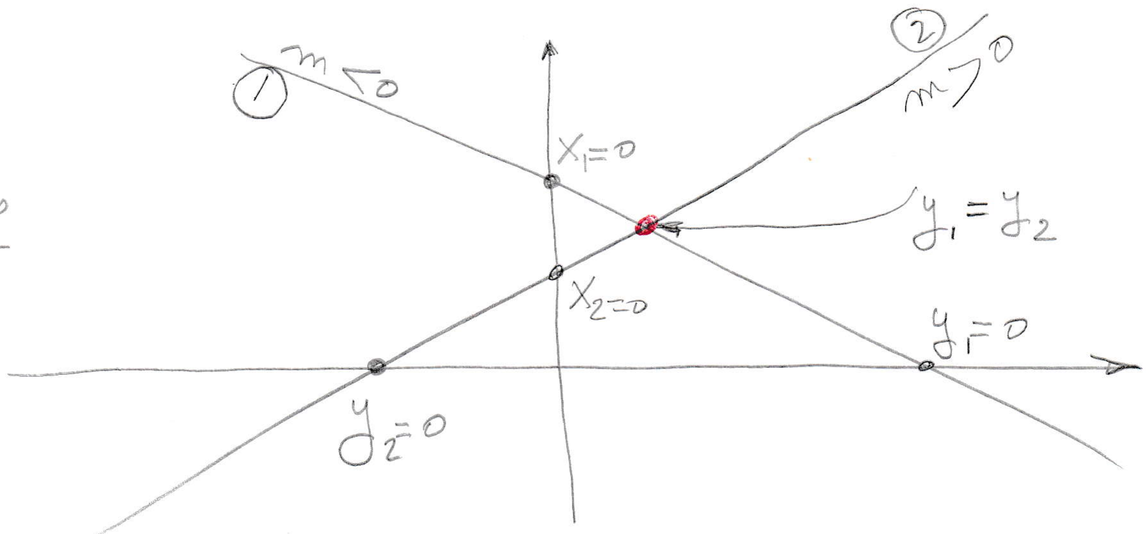


* $m > 0$



* Two lines



Intersection Point $y_1 = y_2$

Linear Inequalities:

Example 4:	$2x + y - 10 \le 0$	can be written as:	$2x + y \le 10$
Example 5:	$2y \le x + 4$	can be written as:	$-x + 2y \le 4$
Example 6:	$2x - 5 \le 3y$	can be written as:	$2x - 3y \le 5$
Example 7:	$-x - y \le 3$	can be written as:	$x + y \ge -3$

Notice that when you multiply by negative, the inequality **is <u>reversed</u>.**

$5 > -3$, multiply by negative: $-5 < 3$

Graphing an Inequality:

Example 8: Graph $2x + y - 5 \le 0$

- Move the constant (-5) , and change to equality:

  $2x + y \le 5$

  $2x + y = 5$

- Isolate y to get two points and to visualize the shape

  $y = -2x + 5$

- Give at least 2 values to x

  $x = 0$, $y = 5$

  $x = 2$, $y = 1$

- Plot the line and decide which half is the solution

Take a point that in <u>Not located</u> on the line and check if it is included in the solution or not.

If it is, then the whole half is included.

For example , take the point (0,0)

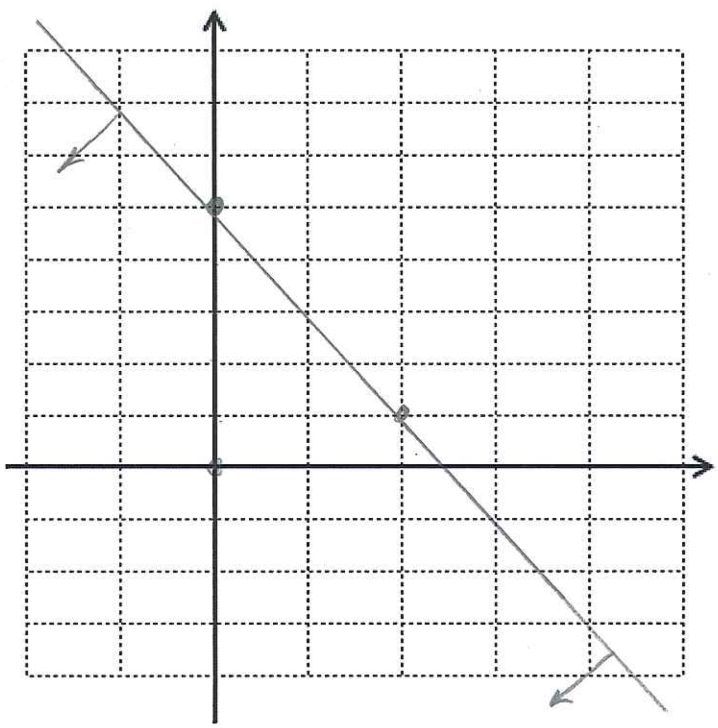
Where x = 0, and y =0

Substitute in step 1:

$2x + y \le 5$

$0 + 0 \overset{?}{\le} 5$

yes



Example 9: Graph the solution set for:

$$3x + 2y - 12 \leq 0 \quad (1)$$

$$-x + 2y \leq 4 \quad (2)$$

$$x \geq 0, \ y \geq 0$$

Find the coordinates of the corner points

- Take each inequality, move the constant if there is , and change to equality
- Isolate y to get two points and to visualize the shape
- Give at least 2 values to x to get 2 points:

$3x + 2y - 12 \leq 0$ (1)	$-x + 2y \leq 4$ (2)
$3x + 2y \leq 12$ $3x + 2y = 12$	$-x + 2y = 4$
$2y = -3x + 12$ $y = \frac{-3x+12}{2}$ X | Y: 0 | 6; 4 | 0 $(0,6), (4,0)$	$2y = x + 4$ $y = \frac{x+4}{2}$ X | Y: 0 | 2; -2 | 1 $(0,2), (-2,1)$

Points:

* A (0,0)

* B: $x = 0$, line 2

$-x + 2y = 4 \Rightarrow y = 2$

B(0, 2)

* C on line (1) & line (2)

$y = y$

$$\frac{-3x+12}{2} = \frac{x+4}{2}$$

$$-4x = -8$$

$$x = 2$$

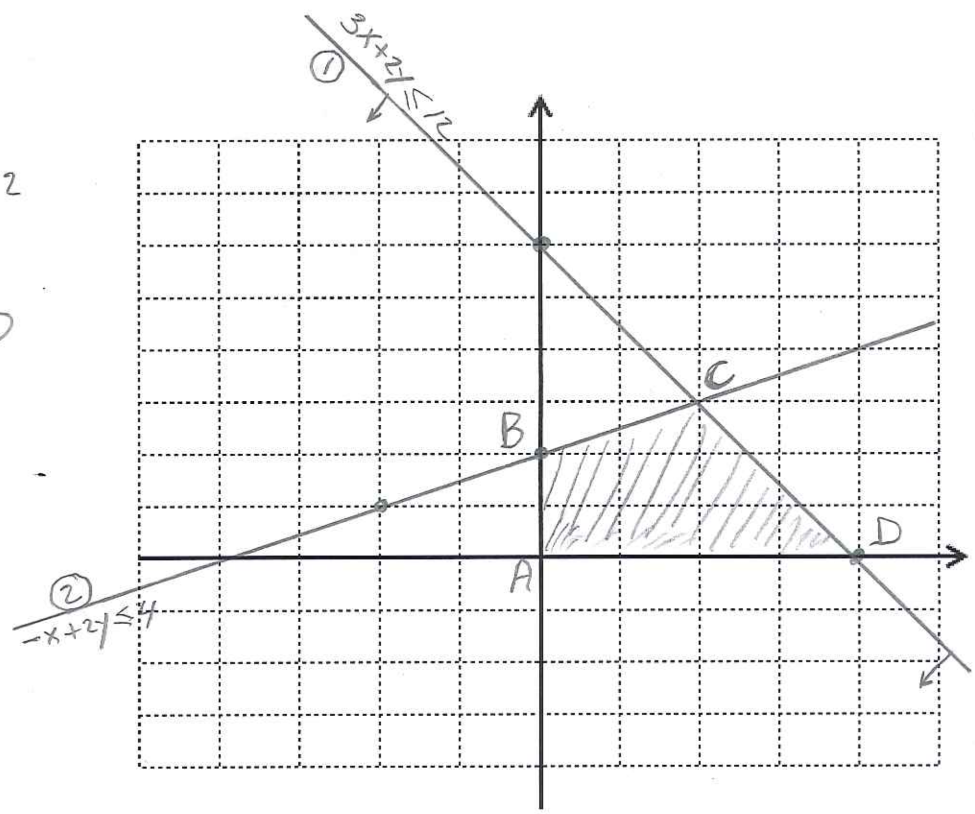
$$y = 3$$

C(2, 3)

* D $y = 0$, on line (1)

$3x + 2y = 12$

$3x = 12 \Rightarrow x = 4$, D(4, 0)



Example 10: Graph the solution set for:

$$-3x + 4y - 6 \le 0 \quad \text{—— ①}$$
$$4x + 3y \ge 9 \quad \text{—— ②}$$
$$x \le 4 \quad \text{—— ③}$$
$$x \ge 0,\ y \ge 0$$

a) Find the coordinates of the corner points.

b) Maximize and Minimize $F = 2x - 4y$

$-3x + 4y \le 6$

$-3x + 4y = 6$

$\boxed{y = \dfrac{3x+6}{4}}$

x	y
2	3
-2	0

①

$4x + 3y \ge 9$

$4x + 3y = 9$

$\boxed{y = \dfrac{-4x+9}{3}}$

x	y
0	3
3	-1

②

$x \le 4$

$x = 4$

③

a) Corner Points

* A: on line ① & line ②, or $y = y$

$$\frac{3x+6}{4} = \frac{-4x+9}{3}$$

Cross Mult → $9x + 18 = -16x + 36$

Solve for x: $x = 0.72,\ y = 2.04$

$A(0.72, 2.04)$

* B: on $x = 4$ and line ①

$-3x + 4y = 6$

$x = 4 \rightarrow y = 4.5$

$B(4, 4.5)$

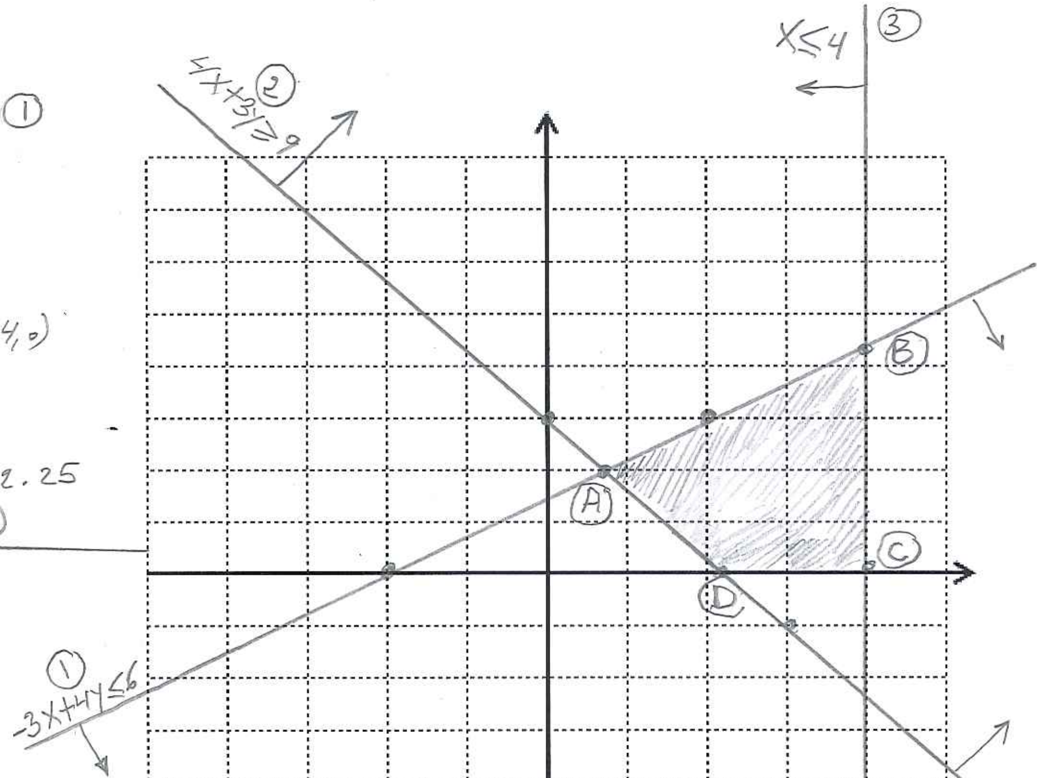
* C: $x = 4,\ y = 0$ $\quad C(4, 0)$

* D: $y = 0$, line ②

$4x + 3y = 9$

$y = 0 \longrightarrow x = 2.25$

$D(2.25, 0)$



b) $F = 2x - 4y$

$A = 2(0.72) + 4(2.04) = -6.72$

$B = 2(4) - 4(4.5) = -10$

$C = 2(4) - 4(0) = 8$

$D = 2(2.25) - 4(0) = 4.5$ $\Longrightarrow$ Max at C, Mini at B