MATH 118 Class Notes For Chapter 5 By: *Maan Omran*

Section 5.1 Central Tendency

- **Mode:** the number or numbers that occur most often.

- **Median:** the number at the midpoint of a ranked data.

Ex1: The test scores for a test were: 78,81,82,76,84,81,76. Find the mode and the median.
- The mode is 81 and 76, both of them repeated twice
- The median must be found after the data is ranked from smallest to largest. For the above data: 76,76,78,81,81,82,84 the median is 81 which is located in the middle.

Ex2: The test scores for a test were: 78,81,82,76,84,86. Find the mode and the median.
- There is no mode, no score is repeated more than once
- The median must be found after the data is ranked from smallest to largest. For the above data: 76,78,81,82,84,86. There are two values in the middle 81 and 82, then the median is average of those two values or (81+82)/2= 81.5

Ex3: The test scores for a test were: 78,78,78,81,81,95,95,95,100. Find the total.
As you noticed, there are repeated scores and it is easier to find the total of those scores this way:
Total = 3(78) + 2(81) + 3(95) + 1(100) = 781

Or: total = $\Sigma f_i x_i$

where: Σ is the symbol for sum

x_i is the score; f_i is the frequency of each score

Ex4: Find the average score for the tests in example 3.
The average score is the total divided by the number of tests, there are 9 tests,
The average is = (781)/9 = 86.78

Or: $\bar{x} = \dfrac{\sum f_i x_i}{n}$

where n is number of tests, $n = \Sigma f_i$ *(sum of frequencies)*

Ex5: A student obtained the following grades for one semester, what is the grade point average GPA?

Course	Grade	Point Value	Frequency (*# of credits*)	Product
Math	A	4	5	20
English	B	3	4	12
Physics	F	0	4	0
Accounting	C	2	3	6
		Sum	16	38

The GPA or the mean is 38/16 = 2.375

<u>Section 5.2</u> Expected Value and Standard Deviation

- **Random Variable:** A function *X* that assigns to every outcome exactly one real number.
- **Probability Density Function:** A list of all possible values of the random variables and the associated probabilities.

Outcomes (events)	Random Variable (X)	Prob. Density (P)
all possibilities	value of each possibility	prob. of each possibility
		Sum = 1

<u>Ex1:</u> An unfair coin in which P(*H*) = 2/3 is flipped twice. The random variable *X* is defined to be the number of heads. Find the density function.

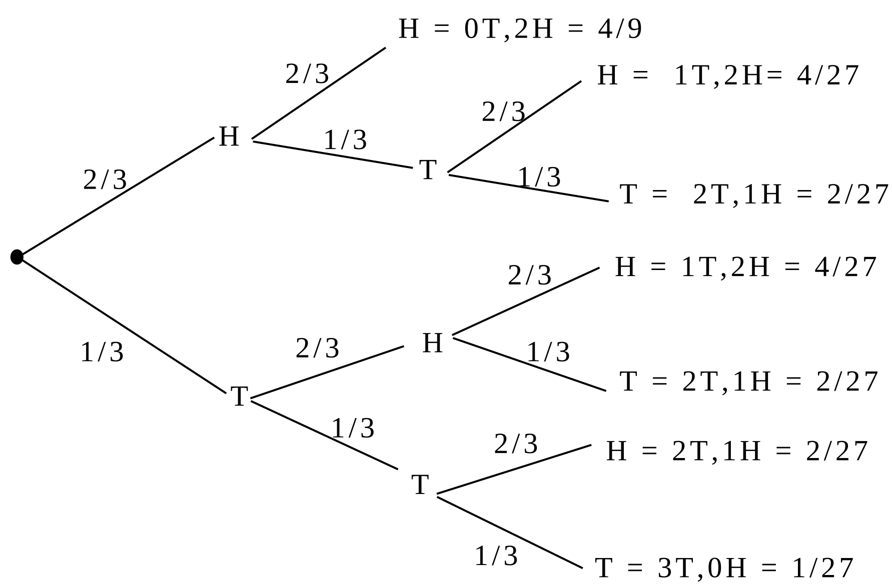
X = the number of heads

Outcomes	X	P
HH	2	2/3·2/3=4/9
HT or TH	1	2/3·1/3 + 1/3·2/3= 4/9
TT	0	1/3·1/3=1/9

- **Using the Tree:**
Use this method when the problem is written is way that the selection is not simple, for example: the experience stops when certain condition is met. We could have used the tree method in example 2 which would make it easier to solve.

<u>Ex2:</u> An experiment consists of flipping an unfair coin where P(*H*) = 2/3 until a total of 2 heads occur or 3 flips. The random variable is defined to be the number of tails. Find the expected value of the random variable.

Outcomes	X	P	X.P
0T, 2 H	0	4/9	0
1T, 2 H	1	4/27 + 4/27 = 8/27	8/27
2T, 1 H	2	2/27 + 2/27 + 2/27 = 6/27	12/27
3T, 0 H	3	1/27	3/27
	Sum	= 1	E(*X*) = 23/27 = 0.85



Binomial & Non-Binomial Distribution ***(Using Tables)***

- **Expected Value E[X], μ:** The mean, the average value of a random variable in which:

  $$\mu = E[X] = \sum P_i X_i = X_1P_1 + X_2P_2 + X_3P_3 + \ ....$$

- **Variance σ^2:** $\sigma^2 = \sum P_i(X_i - \mu)^2$
- **Standard Deviation:** $\sigma = \sqrt{\sigma^2}$

Binomial Distribution ***(Using Formula)***

- **Expected Value $E[X]$, μ:** $\mu = n{\cdot}p$
- **Variance σ^2:** $\sigma^2 = n{\cdot}p.q$
- **Standard Deviation:** $\sigma = \sqrt{\sigma^2}$

Binomial Distribution: A distribution that describes the probability of all possible outcomes of a a series is identical to every other and has two possible outcomes. *(Bernoulli trials of Sec. 4.4)*

$$\boxed{\mathbf{P} = C(n,r).\ p^r.\ q^{n-r}}$$

Ex3: Stereo speakers manufactured with probability of 20% being defective. Three are selected off continuous assembly line, define the random variable X as the number of the defective parts. Find:

a) the density function and the expected value for the defective parts
b) the expected value for the good parts
c) the variance, the standard deviation.

---

a) *X = number of defective parts*

Outcomes	X	P	$X.P$
0D, 3G	0	$C(3,0).(0.80)^3.(0.20)^0 = 0.51$	0
1D, 2G	1	$C(3,1).(0.80)^2.(0.20)^1 = 0.38$	0.38
2D, 1G	2	$C(3,2).(0.80)^1.(0.20)^2 = 0.096$	0.192
3D, 0G	3	$C(3,3).(0.80)^0.(0.20)^3 = 0.008$	0.024
		Sum = 1	= 0.6

Expected value for the defective part is = 0.6

b) Expected value for the good part is = 3 - 0.6 = 2.4

c)

X	P	$(X_i - \mu)$	$(X_i - \mu)^2$	$P_i(X_i - \mu)^2$
0	0.51	(0-0.6) =-0.6	$(-0.6)^2$=0.36	0.51(0.36)= 0.184
1	0.38	(1-0.6) = 0.4	$(0.4)^2$=0.16	0.38 (0.16)= 0.061
2	0.096	(2-0.6) = 1.4	$(1.4)^2$=1.96	0.096 (1.96)= 0.19
3	0.008	(3-0.6) = 2.4	$(2.4)^2$=5.76	0.008 (5.76)= 0.05
			Sum =	0.48

Variance σ^2: $\sigma^2 = \sum P_i(X_i - \mu)^2 = 0.48$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.48} = 0.69$

Ex4: Solve example 3 again but without tables.
This problem is Binomial, first find *n,p & q:* $n = 3$, $p = 0.2$, $q = 0.8$

Expected Value: $\mu = n \cdot p = 3(0.2) = 0.6$ for the defective parts

Variance: $\sigma^2 = n \cdot p \cdot q = 3(0.2)(0.8) = 0.48$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.48} = 0.69$

Ex5: A box with 6 good parts and 4 defective in which 3 are selected. The random variable X is defined as the number of defective parts selected. Find:

a) the density function and the expected value for the defective parts
b) the expected value for the good parts
c) the variance, the standard deviation.

---

a) This problem is not Binomial.

X = number of defective parts

Outcomes	X	P	$X.P$
0D,3G	0	$\frac{C(4,0).C(6,3)}{C(10,3)} = \frac{20}{120}$	0
1D,2G	1	$\frac{C(4,1).C(6,2)}{C(10,3)} = \frac{60}{120}$	$\frac{60}{120}$
2D,1G	2	$\frac{C(4,2).C(6,1)}{C(10,3)} = \frac{36}{120}$	$\frac{72}{120}$
3D,0G	3	$\frac{C(4,3).C(6,0)}{C(10,3)} = \frac{4}{120}$	$\frac{12}{120}$
	Sum	= 1	= 1.2

Expected value for the defective part is = 1.2

b) Expected value for the good part is = 3 - 1.2 = 1.8

c)

X	P	$(X_i - \mu)$	$(X_i - \mu)^2$	$P_i(X_i - \mu)^2$
0	$\frac{20}{120}$	(0-1.2) =-1.2	$(-1.2)^2$=1.44	$\frac{20}{120}$(1.44)= 0.24
1	$\frac{60}{120}$	(1-1.2) = -0.2	$(-0.2)^2$=0.44	$\frac{60}{120}$ (0.44)= 0.02
2	$\frac{36}{120}$	(2-1.2) = 0.8	$(0.8)^2$=0.64	$\frac{36}{120}$ (0.64)= 0.192
3	$\frac{4}{120}$	(3-1.2) = 1.8	$(1.8)^2$=3.24	$\frac{4}{120}$ (3.24)= 0.108
			Sum =	0.56

Variance σ^2**:** $\sigma^2 = \sum P_i(X_i - \mu)^2 = 0.56$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.56} = 0.748$

Ex6: A class with 200 males and 100 females. One student was selected and replaced and that was repeated 6 times. Find:

a) the probability that 2 men will be selected.
b) the expected value for men
c) the expected value for women
d) the variance, the standard deviation.

This problem is a Binomial, find *n,p & q:* $n = 6$, $p = 2/3$, $q = 1/3$.

a) $P = C(6,2).(2/3)^2.(1/3)^4$

b) Expected Value: $\mu = n{\cdot}p = 6(2/3) = 4$ for men

c) Expected Value: $\mu = n{\cdot}p = 6(1/3) = 2$ for women

d) Variance: $\sigma^2 = n{\cdot}p{\cdot}q = 6(2/3)(1/3) = 1.33$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{1.33} = 1.15$

Ex7: A multiple-choice test contains 10 questions with 4 choices for each answer. If a student guesses the answers, find:

a) the probability that he will get 4 correct answers.
b) the expected value for the correct answers
c) the expected value for the wrong answers
d) the variance, the standard deviation.

This problem is a Binomial, find *n,p & q:* $n = 10$, $p = 1/4 = 0.25$, $q = 0.75$.

a) $P = C(10,4).(0.25)^4.(0.75)^6$

b) Expected Value: $\mu = n{\cdot}p = 10(0.25) = 2.5$ for the correct answers

c) Expected Value: $\mu = n{\cdot}p = 10(0.75) = 7.5$ for the wrong answers

d) Variance: $\sigma^2 = n{\cdot}p{\cdot}q = 10(0.25)(0.75) = 1.875$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{1.875} = 1.37$

Ex8: By rolling a pair of dice, a game is played in which you win \$2 if the sum is 2, 3, 4 or 5. You win \$3 if the sum is 6, 7 or 8. You loose \$5 if the sum is 9, 10, 11 or 12. If you pay \$2 to play the game, find the expect gain or loss.

Outcomes	*X*	P	*X*.P
2,3,4,5	+2	1/36+2/36+3/36+4/36=10/36	20/36
6,7,8	+3	5/36+6/36+5/36=16/36	48/36
9,10,11,12	-5	4/36+3/36+2/36+1/35=10/36	-50/36
	Sum	1	E(*X*)=\$ 0.5

Expected gain is \$0.5, but you paid \$2 to play the game, then there is a loss of \$1.5

Section 5.3 Normal Random Variable

$$Z = \frac{X - \mu}{\sigma}$$

Where Z is the Z-score:

Ex1: Let Z be a random variable with normal distribution. Using the table, find:

a) $P(Z \leq 1.87)$
b) $P(0.49 \leq Z \leq 1.75)$
c) $P(-1.77 \leq Z \leq 2.53)$
d) $P(Z \geq 1.87)$
e) $P(-1.66 \leq Z \leq -1.00)$
f) $P(0.00 \leq Z \leq 2.17)$
g) $P(-1.76 \leq Z \leq 0)$

Ex2: Suppose that for a certain population the birth weight of infants in pounds is normally distributed with mean 7.75 pounds and standard deviation of 1.25 pounds. Find the probability that an infant's birth is more than 9 pounds.

Ex3: Bolts produced by a machine are acceptable provided that their length is within the range 5.95 to 6.05 inches. Suppose that the length of the bolts produced are normally distributed with mean of 6 inches and standard deviation of 0.02 inches. what is the probability that a bolt will be an acceptable length?

Ex4: The distribution of low daily temperature in winter in central Florida is random variable with a mean of 50 degrees and a standard deviation of 8 degrees. If the daily temperature falls below 30 degrees, the orange crop will suffer frost damage. What is the probability that on a winter day the orange crop will suffer frost damage?

Ex5: The number of daily order received by mail order firm is normally distributed with a mean 250 and standard deviation 20. The company must either hire extra help or pay overtime on those days when the number of order received is 300 or higher. What percentage of these days must the company either hire extra help or pay overtime?

Section 5.4 Normal Approximation To The Binomial

RULES: To approximate **binomial** probability by normal curve area:

Step 1) determine n, p, q

Step 2) check that both $n.p > 5$ and $n.q > 5$

Step 3) find the expected value and the standard deviation

$$\mu = n \cdot p \qquad \sigma = \sqrt{n.p.q}$$

Step 4) find the new points by:

* subtracting 0.5 from the starting point
* adding 0.5 to the finish point

examples:

$P(3 \leq X \leq 6)$	*will be*	$P(2.5 \leq X \leq 6.5)$
$P(X = 7)$	*will be*	$P(6.5 \leq X \leq 7.5)$
$P(X \geq 8)$	*will be*	$P(X \geq 7.5)$
$P(X \leq 8)$	*will be*	$P(X \leq 8.5)$

Step 5) find the Z-scores and the area under the normal curve using the table

Example: According to the Department of Health and Human Services, the probability is about 80% that a person aged 70 will be alive at the age of 75. Suppose that 500 people aged 70 are selected at random. Find the probability that:

a) exactly 390 of them will be alive at the age of 75

b) between 375 and 425 of them will be alive at the age of 75.

---

a) Step 1) n = 500, p = 0.8, q = 0.2

Step 2) check if both $n.p$ and $n.q$ are more than 5:

$n.p$ = (500).(0.8) = 400

$n.q$ = (500).(0.2) = 100

Step 3) find the expected value and the std. deviation:

$\mu = n \cdot p = (500).(0.8) = 400$

$\sigma = \sqrt{n.p.q} = \sqrt{(500).(0.8).(0.2)} = 8.94$

Step 4) find the new point:

P(X = 390) will be P(389.5 $\leq X \leq$ 390.5)

Step 5) find the Z-score:

X = 389.5, $Z = \dfrac{389.5 - 400}{8.94} = -1.17$

X = 390.5, $Z = \dfrac{390.5 - 400}{8.94} = -1.06$

and now by using the table:

P(-1.17 $\leq Z \leq$ -1.06) = 0.1446 - 0.1210 = 0.0236

b) for P(375 $\leq X \leq$ 425), we use the information of steps 1, 2 and 3 then:

P(375 $\leq X \leq$ 425) will be P(374.5 $\leq X \leq$ 425.5)

X = 374.5, $Z = \dfrac{374.5 - 400}{8.94} = -2.85$

X = 425.5, $Z = \dfrac{425.5 - 400}{8.94} = 2.85$

and now by using the table:

P(-2.85 $\leq Z \leq$ 2.85) = 0.9978 - .0022 = 0.9956

Note: If you try to find P(X=390) of the first question by applying the Bernoulli formula, you will have:

C(500,390).$(0.8)^{390}$.$(0.2)^{110}$ which cannot be found by most calculators, and that is why the approximation method is very useful.