

Sections 4.1, 4.2

1. Five students, Ann, Bill, Chuck, Debra and Ed are randomly assigned seats from the seats numbered 1 through 10. What is the probability that Ann is assigned either #1 or #10?
2. A pair of dice are rolled and the number of dots on the top are noted. Assume both dice are fair, what is the probability that:
 - a) the numbers are the same on the two dice?
 - b) the sum of the numbers is even?
3. A bucket contains 10 audio taped in which 3 are defective. Three tapes are selected at random, what is the probability that:
 - a) at most one is defective?
 - b) at least one is defective?
4. A sales person plans to visit Indianapolis, Chicago, Detroit, Toledo and Buffalo on a business trip. If his boss randomly selects the order of the 5 cities to be visited, what is the probability that:
 - a) Indianapolis will be last
 - b) Detroit and Toledo after each others in either order

Answers (not in order):

[a) 98/120 b)17/24] ; [2. P(9,4)/P(10,5) = 1/5] ; [a) 1/5, b) 2/5] ; [a) 1/6 b) 18/36] ; [1/2] ;

- 5) Let $P[A \cup B] = 0.9$, $P[B] = 0.75$. Find $P[A]$ if:
 - a) A and B are independent events.
 - b) A and B are disjoint events.
 - c) B is contained by A
6. Let A , B , C and D be events which form partition of a sample space S . If $P[A]=P[B]$, $P[C]=2P[A]$, and $P[D]= 2P[C]$. Find: $P[A]$, $P[A \cup B]$
7. Suppose E , F and G are events in a sample space S with $P[E] = 0.7$, $P[F] = 0.4$, and $P[G] = 0.2$, $P[E \cup F] = 1$, and $F \cap G = \emptyset$. Find $P[E \cap G]$.
8. Suppose A and B are events in a sample space S with $P[A] = 0.5$, $P[B] = 0.3$. If $B \subset A$, find $P[A \cup B]$
9. Let C and D be events with $\Pr[C]=0.4$ and $\Pr[D]=0.5$ and $\Pr[C \cup D]=0.6$. Find $\Pr[C|D]$.
10. Let A and B be events in a sample space S with $P[A] = 0.6$, $P[A \cap B'] = 0.2$, and $P[B] = 0.5$. Find $P[A/B]$.
11. The events E and F are independent in a sample space S , $P[E]=0.3$, and $P[F]=0.6$. Find $P[E/F]$.
12. Let A and B be events such that $P[A \cup B] = 0.8$ and $P[A]=0.6$. What is the $P[B]$ if A and B are independent?
13. Let A and B be events such that $P[A] = 0.6$, $P[A \cup B] = 0.9$, and $P[B/A]=0.4$. Find $P[B]$.

Answers #5 - 13 (not in order):

[0.8] ; [0.375] ; [0.5] ; [0.2] ; [0.6, 0.15, 0.9] , [3/5], [1/8, 1/4] ; [0.3] ; [0.54] ; [0.5]

14. A student attends mathematics class with probability 0.6, skips accounting class with probability 0.3, and attends both with probability 0.5. Find the probability that she attends at least one class.
Answer: 0.8

15. Half of student attending the business class read the *Wall Street Journal*, and $\frac{2}{3}$ of the students read *The Indianapolis Star*. Reading those newspapers are independent, what is the probability that a randomly selected student read exactly one of these newspapers?
Answer: 1/2

16. A television set which sometime malfunctions has the following probabilities:
A clear picture = 0.7
Good color = 0.6
Good sound = 0.5
Both clear picture and good sound = 0.4
Both clear picture and good color = 0.5
Both good sound and good color = 0.4
All = 0.3

Find the probability of exactly one of the three characteristics.

Answer: 0.1

17. An urn contains 8 red, 6 blue and 7 white marbles. You reach in and randomly select 5 marbles without replacement. Find the probability that you will get:
a) exactly 2 red
b) same color
c) more than one color
d) at least 1 red

Answer: a) 0.394 b) 0.00408 c) 0.9959 d) 0.9368

18. There are 2 white balls, 2 red balls, and 3 blue balls in a box. Two balls are selected and their colors are noted. Find the probability that neither is white given that neither is blue.
Answer: 1/6

19. A box contains 1 green, 2 blue, and 2 red balls. Two balls are selected and their colors are noted. Find the probability that one is red and one is blue given that they are not same color.
Answer: 1/2

20. A team of 5 people will be selected out of 10 men and 12 women.
a) Find the probability that the team include both genders.
b) Find the probability that the team include all women, given they are same genders
Answer: a) 0.96 b) 0.7586

21. A pair of dice are rolled and the number of dots on the top are noted. Assume both dice are fair, what is the probability that:
d) at least one of the numbers is even given that the sum is 8?
e) exactly one die shows 4 given that the sum of the numbers is 7?
Answer: a) 3/5 b) 1/3

Sections 4.3, 4.4

22. A firm has new test to detect lung cancer. It researches the values of the test on thousands of people and find that if a person has lung cancer, then the test will detect it in 97% of the cases (*correct positive*); and if a person does not have lung cancer, it will still show a positive result in 2% of the cases (*false positive*). Later medical records show that 9% of the group actually did have lung cancer.

What is the probability that a person who tests:

- positive actually has lung cancer?
- positive actually does not have lung cancer?
- negative actually has lung cancer?

Answer: a) $0.8275 = 82.75\%$
b) 17.25%
c) $0.003 = 0.3\%$

23. Repeat question 22. What is the probability that a person who:

- has lung cancer, tests positive?
- does not have cancer, tests positive

Answer: a) 97%
b) 2%

*24. Consider 2 bags, each of which contains 3 colored balls. Bag *a* contains 2 red and 1 white ball, and bag *b* contains 1 red, 1 white, and 1 blue ball. An experiment consists of randomly drawing ball from bag *a*, noting its color, placing the ball in bag *b*, and then randomly drawing a ball from bag *b* and noting its color. Draw the tree diagram and find the probability of obtaining 2 red balls.

Answer: $1/3$

25. A new small business makes a profit the first year with probability 0.2. After the first year it makes a profit with probability 0.6 if it made a profit in the preceding year, and it makes a profit with probability 0.4 if it did not make a profit in the preceding year. Find the probability that it makes a profit for exactly 2 of the first 3 years.

Answer: 0.272

26. A box contains 1 red, 2 green, and 3 yellow balls. A ball is selected at random and its color is noted. If it is green it is replaced; otherwise it is not. A second ball is selected and its color is noted. If the second is green, find the probability that the first was green also.

Answer: $5/17$

27. A selection is made between two coins and the selected coin is flipped twice. One of the coins is fair and the other is unfair with $P[H] = 2/3$. If the result of the experiment is one head and one tail, what is the probability that the unfair coin was selected?

Answer: $8/17$

28. We flip a fair coin. If the coin shows heads, we draw a marble from urn H with 3 red, 4 white and 1 blue marble. If the coin shows tails, we draw a marble from urn T with 4 red and 2 blue marbles.

- Find the probability that the the marble chosen was red , given that the coin showed tails.
- Find the probability that the coin showed tails, given that the marble chosen was blue

Answer: a) $2/3$
b) $8/11$ or 0.727

29. An unfair coin with $\Pr[H] = 0.6$ is flipped. If the flip results in a head, one student is selected at random from a class of 6 boys and 4 girls. If the flip results in a tail, one student is selected from a different class containing 5 boys and 7 girls.

- a) Find the probability that a boy is selected
- b) Find the probability that a girl is selected given that the coin shows heads.

Answer: a) 0.527
b) $2/5$ or 0.4

30. Repeat question 29, If a girl was selected, find the probability that the coin showed tail

Answer: 0.493

31. There are four coins in your pocket; three coins are fair and one is two-headed. You randomly select a coin and flip it. What is the probability of getting heads on the flip?

Answer: $5/8$

32. In a lab, there are 8 mice: 1 gray male, 4 white females, and 3 white males. Two mice are selected one after the other without replacement and their color and sex are noted, find the probability that:

- a) the first is was a female given the second was a male
- b) the first is was a gray given the second was a male

Answer: a) $4/7$ b) $3/28$

33. A multiple-choice test contains 12 questions, with 4 choices for each answer. If a student guesses the answers, find the probability that he will get at least 1 correct answer.

Answer: $1 - [C(12,0)(0.25)^0(0.75)^{12}] = 0.9683$

34. A fair die is rolled 10 times. The roll is considered a success if the die shows a "4". Find the probability that there will be:

- a) exactly 2 successes in 10 rolls
- b) at least 1 success

Answer: a) 0.2907 b) 0.8385

35. An unfair coin with $\Pr[\text{heads}] = 0.7$ is flipped five times.

- a) What is the probability of getting exactly three heads in the five flips?
- b) What is the probability of getting four or more heads in five flips?
- c) What is the probability of getting at least one tail?

Answers: a) 0.3087 b) 0.52822 c) 0.83193

36. In a city 50 % of the voters are Democrats, 25 % are independent, and 25% are Republican. If 10 people are asked at random, what is the probability that at least 7 are Democrats?

Answer: 0.171