# Bonus, Due:  Sunday , December 1, 11:00 PM (NO DELAY)

The homework is worth 10 points.   You must show your work for credits. Solving theses questions will greatly help you understand 9.1 and 9.2. The more you understand the homework, the easier chapter 9 will be.

**IMPORTANT**: **Keep this handout, (Do Not show work within the handout. Use separate papers**) scan or take clear photo of your work and e-mail it to me for credits. The answers are given, so you must show your work.

1)   If: $T = \begin{vmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{vmatrix}$ .  Find:      (*see the following page for hints*).

   a) $T^2$         b) $T^3$         c) $T^4$         d) $T^5$

2)   If: $P = |0.3 \quad 0.7|$ and $T = \begin{vmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{vmatrix}$ . Use the results of question 1 to multiply:

   a) $P.\ T$         b) $P.\ T^2$         c) $P.\ T^3$         d) $P.\ T^4$

3)   If: $T = \begin{vmatrix} 0.1 & 0.3 & 0.6 \\ 0.2 & 0.4 & 0.4 \\ 0 & 0.1 & 0.9 \end{vmatrix}$ .  Find:

   a) $T^2$         b) $T^3$         c) $T^4$

4)   If: $P = |0.2 \quad 0.3 \quad 0.5|$ and $T = \begin{vmatrix} 0.1 & 0.3 & 0.6 \\ 0.2 & 0.4 & 0.4 \\ 0 & 0.1 & 0.9 \end{vmatrix}$ Use the results of question 3 to multiply:

   a) $P.\ T$         b) $P.\ T^2$         c) $P.\ T^3$         d) $P.\ T^4$

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*Answers*:  (Notice the sum per row = 1 in all original matrices and also in the results). *(One answer does not completely match, can you highlight in your work)*.

1)   a) $\begin{vmatrix} 0.520 & 0.480 \\ 0.360 & 0.640 \end{vmatrix}$   b) $\begin{vmatrix} 0.392 & 0.608 \\ 0.456 & 0.544 \end{vmatrix}$   c) $\begin{vmatrix} 0.443 & 0.557 \\ 0.418 & 0.582 \end{vmatrix}$   d) $\begin{vmatrix} 0.423 & 0.577 \\ 0.433 & 0.567 \end{vmatrix}$

2)   a) $|0.48 \quad 0.52|$   b) $|0.408 \quad 0.592|$   c) $|0.4368 \quad 0.5632|$   d) $|0.4253 \quad 0.5747|$

3)   a) $\begin{vmatrix} 0.07 & 0.21 & 0.72 \\ 0.1 & 0.26 & 0.64 \\ 0.02 & 0.13 & 0.85 \end{vmatrix}$   b) $\begin{vmatrix} 0.049 & 0.177 & 0.774 \\ 0.062 & 0.198 & 0.740 \\ 0.028 & 0.143 & 0.829 \end{vmatrix}$   d) $\begin{vmatrix} 0.040 & 0.163 & 0.797 \\ 0.046 & 0.172 & 0.782 \\ 0.031 & 0.149 & 0.820 \end{vmatrix}$

4)   a) $|0.08 \quad 0.23 \quad 0.69|$         b) $|0.054 \quad 0.185 \quad 0.761|$

   c) $|0.0424 \quad 0.1663 \quad 0.7913|$         d) $|0.0375 \quad 0.15837 \quad 0.80413|$

The following example will be helpful in Markov Chain sections 9.1 and 9.2.

If: $A = \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix}$  find $A^2$, $A^3$, $A^4$ and $A^5$

$$A^2 = A.A = \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix}$$

$$A^3 = A^2.A = \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix} \cdot \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = \begin{vmatrix} -3 & 1 \\ -2 & -2 \end{vmatrix}$$

$$A^4 = A^2.A^2 = \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix} \cdot \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 3 \\ -6 & 2 \end{vmatrix}$$

$$A^5 = A^2.A^3 = \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix} \cdot \begin{vmatrix} -3 & 1 \\ -2 & -2 \end{vmatrix} = \begin{vmatrix} 5 & 1 \\ -2 & 6 \end{vmatrix}$$