# Disorder can eliminate oscillator death

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We use an array of diffusively coupled limit-cycle oscillators with a regular monotonic trend of natural frequencies to demonstrate that the disorder introduced in the form of random deviations from a linear trend of frequencies can weaken considerably desynchronization-induced oscillator death and, as a result, increase oscillation intensity substantially. There exist definite optimal levels of magnitude of spatial disorder at which maximal oscillatory energy is attained in the array.

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## I. INTRODUCTION

Synchronization of arrays of oscillatory systems has always been a subject of inquiry because they are frequently encountered in physics, engineering, chemistry, biology, and other branches of science. Modern applications include chains of lasers, Josephson junctions, and relativistic magnetrons, as well as modeling of the mechanisms of rhythmic activity of cardiac, nervous systems, and others (see, e.g., the literature cited in [1,2]). In recent years this interest was also associated with the advance made in constructing information processing systems consisting of a large number of active elements (cellular neural networks [3]). In this connection, of special interest is the influence of inhomogeneities on synchronization in oscillator lattices. This influence proved to be highly nontrivial. Inhomogeneities (including spatially irregular ones, i.e., disorder) introduced into a system in which complex spatiotemporal patterns exist can, for example, lead to more synchronous behavior of the oscillators. Examples include improved synchronization in ensembles of coupled nonlinear pendulums modeling chains of Josephson junctions [4] and in arrays of coupled maps used as models of earthquake dynamics [5], as well as regularization of dynamics in chains of coupled chaotic oscillators [6]. Spatially uncorrelated noise can enhance stochastic resonance effects in the spatiotemporal variant [7], facilitate signal propagation in arrays of bistable systems [8], sustain traveling waves in subexcitable chemical media [9], sustain patterns (including spiral ones) [10], induce pattern transitions [11] and fronts [12], and so on.

However, the influence of spatial disorder on oscillator death in oscillator arrays with local couplings was outside the scope of the references cited above. The phenomenon of oscillator death has attracted the attention of researchers since the appearance of Ref. [13]. Significant progress was achieved in theoretical and numerical analysis of oscillator death in systems of oscillators with ''all-to-all'' coupling [1,14]. In recent years oscillator death has become an object of experimental study [15]. Our paper deals with oscillator death in oscillatory arrays with local couplings, namely, in chains of self-oscillators [16,17], and we study here the action of disorder on such an oscillator death. During such an oscillator death, which occurs at sufficiently strong dissipative coupling in regions of fast increase (or decrease) of natural frequencies along the chain, regions of vanishing oscillation amplitude are formed, even if the conditions of self-excitation are fulfilled for each element in the absence of coupling. The mechanism of formation of such regions is based on increased losses of oscillations in each oscillator under the action of sufficiently large dissipative coupling after breakdown of synchronization [17]. An example of synchronized clusters and regions of oscillator death in an array of $10^3$ coupled oscillators is given in Fig. 1 [18]. In this example we use the equations for slow complex amplitudes $z_j=|z_j|\exp(i\varphi_j)$ in the form

$$\dot{z}_j=i\omega_j z_j+(p-|z_j|^2)z_j+d(z_{j+1}-2z_j+z_{j-1}). \qquad (1)$$

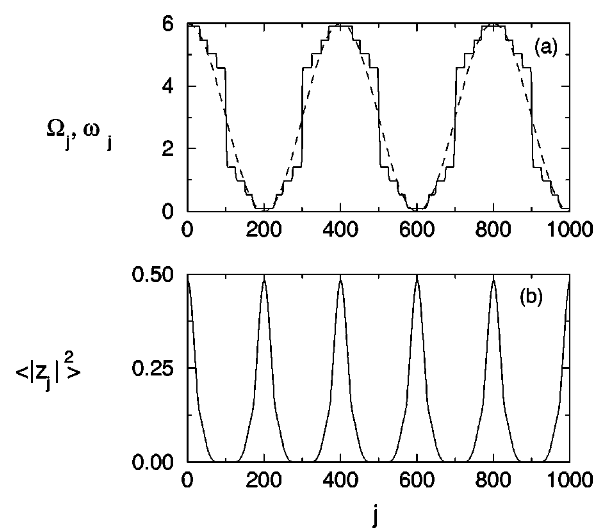


FIG. 1. (a) The dashed line shows the spatial distribution of natural frequencies $\omega_j$ in the model (1). The solid line shows the averaged frequencies of oscillations $\Omega_j$ of the elements of the array (1) for such $\omega_j$. (b) Spatial profile of time-averaged intensity of oscillations $\langle|z_j|^2\rangle$. One can easily see regions of synchronized clusters of oscillations (where the spatial variations of natural frequencies $\omega_j$ are relatively slow—in the vicinity of the extrema of cosinelike distribution of $\omega_j$ in space) and regions of oscillator death with infinitely small amplitude of oscillations (where the gradient of $\omega_j$ is relatively large).

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Here, the diffusive coupling constant $d=20p$ and the range of frequency scatter $\Delta\omega=12p$, where $p=0.5$ is the oscillation growth rate.

At first sight it may seem that introduction into the system of additional inhomogeneities by superimposing random frequency changes on the smooth frequency trend will only enhance the effect of oscillator death. However, we found that the inverse effect is also possible. Under definite conditions, the degree of death caused by the monotonic trend of natural frequencies can be decreased substantially by introducing spatial disorder. There exist optimal levels of dispersion of spatial disorder at which maximal values of mean ''incoherent'' energy $\varepsilon\sim\langle\Sigma_{j=1}^{N}|z_j|^2\rangle$ and mean ''coherent'' energy $w\sim\langle|\Sigma_{j=1}^{N}z_j|^2\rangle$ are attained in the array.

In the next section we present the model we use to study the action of disorder on oscillator death. Section III is devoted to our numerical experiments and discussion of their results. In Sec. IV we suggest an explanation of the phenomena observed in numerical experiments. Finally, we end with a brief summary of this work and our conclusions.

## II. MODEL DESCRIPTION

In this work we restrict ourselves to considering a one-dimensional chain of diffusively coupled oscillators with free ends and linear variation of frequency along the chain. Such a model was chosen for the following reasons. On the one hand, this model provides a rather broad range of parameters in which synchronized clusters coexist with regions of oscillator death as is typical of inhomogeneous systems. On the other hand, the model contains the simplest type of inhomogeneity in the sense that, in the limit when the phase approximation [19] is valid, this system, with end effects neglected, becomes homogeneous because the meaningful parameter in this limit is not the frequency but the frequency difference between neighboring elements. This simplicity allows one in a number of cases to obtain the simplest scaling approximations and formulate fairly general results employing a limited number of numerical solutions [17].

In addition, analysis of such chains is of independent interest because they arise in a natural manner when phenomena observed in real life are modeled. Two most instructive examples are the dynamics of the narrow intestine of mammals and vortex shedding in a flow behind cone-shaped bodies (e.g., supports or chimneys). If the small intestine is divided into sections 1–3 cm long, then each of them is able to oscillate at a definite frequency that changes along the intestine almost linearly over large enough distances [20]. Investigation of the linear changes of frequency observed at vortex shedding also involves analysis of chains of coupled oscillators with linearly varying natural frequencies, if derivatives with respect to the coordinate along the cone axis are replaced by finite differences (see, e.g., [21]).

The analysis is carried out on an example of chains of oscillators whose dynamics in a quasiharmonic approximation is described by Eq. (1) for slowly varying complex amplitudes $z_j$ with the boundary conditions $z_1=z_0$; $z_{N+1}=z_N$ corresponding to a chain with free ends. The set of equation (1) is the normal form for a chain of diffusively coupled oscillators of a general form in the neighborhood of a Hopf bifurcation. In subsequent numerical experiments we set $p=0.5$ and $d=20p$ (unless otherwise specified), and the number of oscillators $N=100$. Here we will consider the influence of random deviation of frequency dependence along the chain from the linear one for three types of frequency distributions that are interesting in terms of applications.

In type I distributions, neither total frequency range nor frequency ensemble were fixed and they could change due to variation of both the regular trend and the range of random scattering. In type II and III distributions, the total range of values of the natural frequencies of oscillators was constant, and only the relative contribution of regular and random components was varied.

Although manifestations of the considered effects in specific applications are outside the scope of our paper, we chose as qualitative characteristics the functionals that can be useful for estimating the action of signals from oscillator chains on some types of sensors. These functionals are the normalized mean ''incoherent'' energy

$$\varepsilon=\frac{\left\langle \sum_{j=1}^{N}|z_j|^2\right\rangle}{\left\langle \sum_{j=1}^{N}|z_j^{(0)}|^2\right\rangle}=\frac{\left\langle \sum_{j=1}^{N}|z_j|^2\right\rangle}{Np} \tag{2}$$

and normalized mean ''coherent'' energy

$$w=\frac{\left\langle \left|\sum_{j=1}^{N}z_j\right|^2\right\rangle}{\left\langle \left|\sum_{j=1}^{N}z_j^{(0)}\right|^2\right\rangle}=\frac{\left\langle \left|\sum_{j=1}^{N}z_j\right|^2\right\rangle}{N^2p}, \tag{3}$$

where $z_j^{(0)}$ are the complex amplitudes of in-phase oscillations excited in the limit of infinitesimal frequency mismatch; $\langle\cdot\rangle$ denotes averaging over time.

## III. EFFECT OF SPATIAL DISORDER ON OSCILLATOR DEATH

For a linear frequency variation along the chain (in the absence of disorder), oscillator death is manifested as formation at the center of the chain of a region in which oscillation amplitudes are vanishing. Oscillator death is associated with the fact that, for a large difference of natural frequencies of neighboring oscillators, the influence of nonresonant terms proportional to $z_{j+1},z_{j-1}$ in the equation for $z_j$ is relatively weak, and the diffusive coupling introduces damping (the term $-2dz_j$) that exceeds amplification at large $d$ ($d\geqslant p/2$). For chains with free ends, when the linear frequency trend grows, this effect is manifested first at the center of the chain where desynchronization occurs first with increasing frequency mismatch [17] (Fig. 2).

### A. Oscillator death elimination by disorder expanding the total frequency range (type I distribution)

For the type I distribution considered in this section, the value of the linear frequency trend $\Delta\omega$ and the range of random frequency scatter $\Delta\omega^*$ changed independently of each other, so that

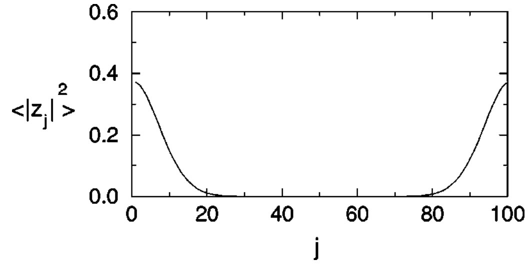


FIG. 2. Oscillator death in the chain with constant frequency gradient (without disorder). Total frequency range (the difference between the lowest and highest frequencies $\omega_1$ and $\omega_{100}$) is 6.0. Plotted is the time-averaged intensity profile.

$$\omega_j = \Delta\omega \frac{j-1}{N-1} + \Delta\omega^* \xi_j , \tag{4}$$

where $\xi_j$ are random numbers distributed uniformly in the interval $[-0.5;+0.5]$ and $N$ is the number of oscillators in the chain.

In typical variants of this series, introduction of disorder into a distribution of natural frequencies either did not significantly affect oscillator death (''unfavorable'' disorder) or resulted in pronounced growth of the oscillator amplitude, so that the region of death diminished markedly or even vanished (''favorable'' disorder). The latter was dominating in the series on the average. Typical examples are represented by spatial distributions of time-averaged oscillation intensities $\langle |z_j|^2 \rangle$ in Fig. 3(a) and in Fig. 4(a), and by spatiotemporal diagrams for $|z|$ in Fig. 3(b) and Fig. 4(b). Figures 3(c) and 4(c) show spatiotemporal diagrams for $\mathrm{Im} z_j(t)$ that illustrate the variation of phase in time and space: the variation of the picture from maximally dark to light in the region of smooth variations of amplitude corresponds to a change of phase by $\pi$.

Mean ''incoherent'' $\varepsilon$ and ''coherent'' $w$ energies are plotted in Fig. 5 versus the range of random frequency scatter of oscillator natural frequencies relative to the linear trend for different values of the trend (the values of $\varepsilon$ and $w$ were obtained by averaging the ''incoherent'' and ''coherent'' energies over the ensemble of 25 sets of natural frequencies $\{\omega_j\}_{j=1}^N$ with different samples of disorder). From the data presented in Fig. 5(a) it follows that, for the values of parameters $\Delta\omega$ and $d$ corresponding to weak manifestation of the effect of oscillator death $\varepsilon(\Delta\omega^*=0)\approx 0.6-0.7$ [the curves in Fig. 5(a) for $\Delta\omega = 0.75,1.5$], introduction of disorder into the frequency distribution does not lead to pronounced changes in the average level of incoherent energy $(\sim 10\%)$. However, for $\Delta\omega$ and $d$ such that $\varepsilon(\Delta\omega^*=0) \approx 0.1 \simeq 0.3$, more than a twofold increase of incoherent energy can occur when disorder is introduced. It is worth noting that this effect is interesting not only from the academic point of view, because it is observed at the values of $\varepsilon$ that are of practical importance. If the effect is evaluated in comparable frequency gradients, then, as follows from the data given in Fig. 5(a), introduction of spatial disorder can be equivalent to more than a twofold decrease of frequency gradient. For example, the oscillator death effect corresponding to $\varepsilon = 0.3$ for $\Delta\omega = 3.0$ is approximately the same as for $\Delta\omega = 6.0$ but with additionally imposed frequency disorder.

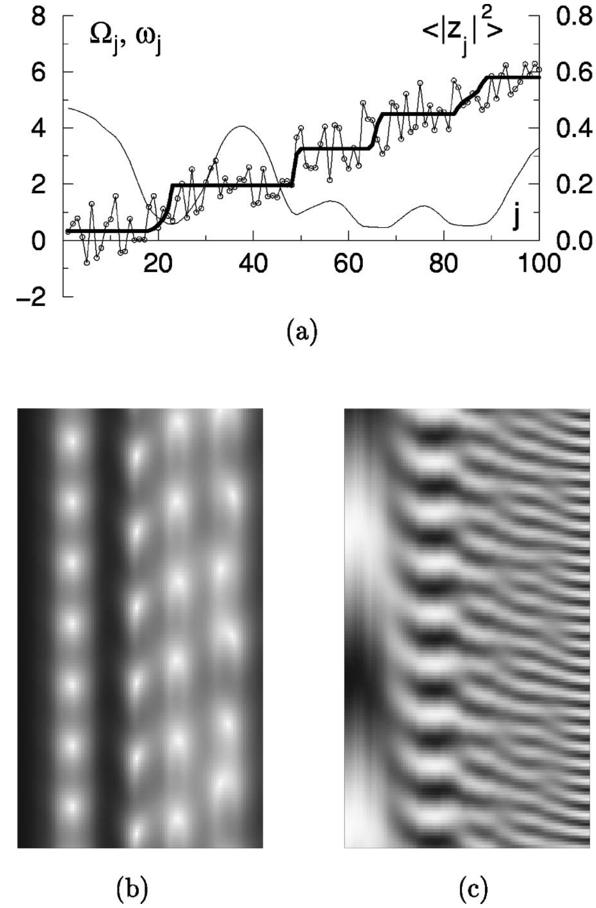


FIG. 3. Chain with linear frequency trend and introduced disorder (4), $\Delta\omega = 6.0$ and $\Delta\omega^* = 0.2\Delta\omega$. (a) Time-averaged intensity profile $\langle |z_j|^2 \rangle$ (solid line), averaged frequencies of oscillations $\Omega_j$ (bold line), and natural frequencies $\omega_j$ (circles connected by dashed line). (b) $x$-$t$ plot of amplitude $|z_j(t)|$ (time increases upward for 30 units and the position along the array varies horizontally); white corresponds to zero and black to the maximal value of the amplitude, which is about $\sqrt{p}$. (c) $x$-$t$ plot of $\mathrm{Im} z_j(t)$; white and black colors correspond to minimal and maximal values of $\mathrm{Im} z_j(t)$ (about $\pm\sqrt{p}$), respectively.

Concerning the ''coherent'' energy $w$ we can say that, even in the case of complete synchronization in a chain with a linear trend, the value of $\langle |\Sigma_{j=1}^N z_j|^2 \rangle$ may be much smaller than its maximum $N^2 p$ because of the finite phase difference between oscillations of neighboring oscillators. Consequently, analysis of the normalized value of $w$ is meaningful not only for $w \approx 1$ but for $w \ll 1$ too. As follows from the data in Fig. 5(b), in this case introduction of disorder into a frequency distribution may have the same effect as decrease of the large-scale frequency gradient by more than three times. In addition, under the action of disorder $w$ can change in a wider interval than $\varepsilon$. For instance, for the data in Fig. 5(b), introduction of disorder at $\Delta\omega = 2.25$–$4.5$ results in an almost fourfold increase of $w$.

A distinguishing feature of the dependences $\varepsilon = \varepsilon(\Delta\omega^*)$, $w = w(\Delta\omega^*)$ shown in Fig. 5 is the existence of the optimal value $\Delta\omega^*_{opt}$ maximizing $\varepsilon$ and $w$. It is interesting that this optimal value of the frequency band $\Delta\omega^*$ characterizing the spread relative to the mean value at each point of the array proved to be comparable to the total range of

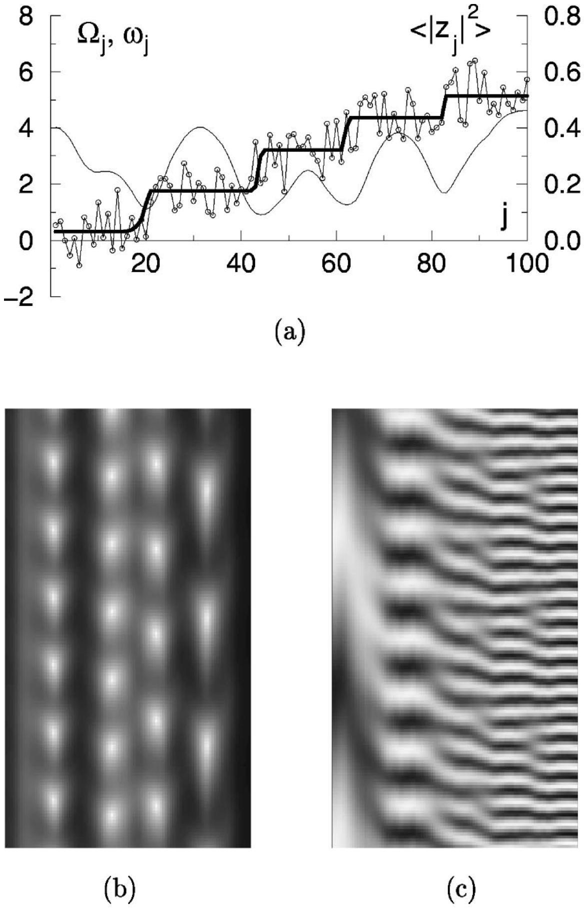


FIG. 4. The same as in Fig. 3 but for another sample of disorder (i.e., for another set of $\{\xi_j\}_{j=1}^{N}$).

large-scale (regular) variation of frequency $\Delta\omega$. At least, this is true for the parameter region of practical interest in which $\varepsilon > 0.1$, $w > 1/N$ in the absence of disorder.

### B. Oscillator death elimination by disorder conserving the total frequency range (type II and III distributions)

In the previous section we considered an idealized situation when large-scale and small-scale inhomogeneities change independently of each other. The case of an ensemble of interacting oscillators formed when definite restrictions are imposed is also interesting in terms of applications. In particular, if inhomogeneities are due purely to design, then one can formulate a problem about the influence of element redistribution in space, with the total range of scatter unchanged.

In the type II distribution that was used earlier in investigations of the influence of random frequency deviations from a linear trend on cluster synchronization [17], we have

$$\omega_j = \omega_0 + \frac{\Delta\omega^*}{2} + \frac{(j-1)(\Delta\omega - \Delta\omega^*)}{(N-1)} + \Delta\omega^* \xi_j, \quad (5)$$

so that the frequency distribution changes from monotonic, completely regular (at $\Delta\omega^* = 0$) to a completely irregular one (at $\Delta\omega^* = \Delta\omega$). Here, again, $\xi_j$ are random numbers uniformly distributed in the interval $[-0.5, +0.5]$, $N$ is the number of elements in the array, $\Delta\omega$ is the total range of natural frequencies of all elements of the array, and $\Delta\omega^*$ is the bandwidth of random variations of frequency. In fact, with such a choice of frequencies, the oscillator ensembles forming the array in each series may appear to be quite different.

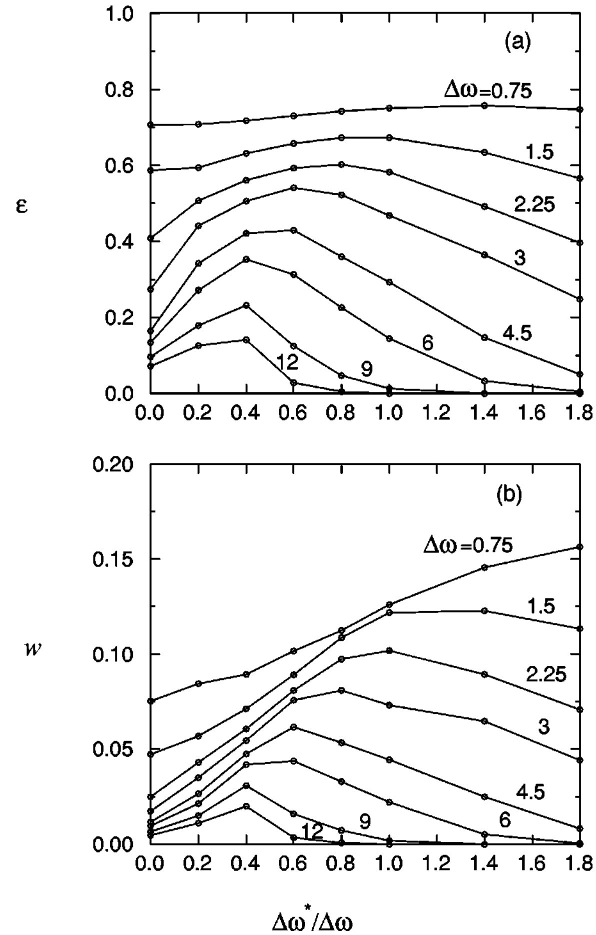


FIG. 5. Dependences of ''coherent'' (a) and ''incoherent'' (b) energies $\varepsilon$ and $w$ on the relative level of disorder $\Delta\omega^*/\Delta\omega$ introduced by the rule (4) for different values of linear frequency trend $\Delta\omega$.

In type III distributions random inhomogeneity was introduced into the linear trend $(j-1)\Delta\omega/(N-1)$ for a constant ensemble of oscillators by their random permutation in the array using the following algorithm. We chose a random number $m$ that equiprobably took the value $1,2,\ldots,[\Delta\omega^*/2\Delta\omega]$, where $[n]$ is the integer part of $n$. Then, the first and $m$th oscillators exchanged places [i.e., $\omega_1 = \Delta\omega(m-1)/(N-1)$, $\omega_m = \Delta\omega(1-1)(N-1) = 0$]. Further, we took the unpermuted oscillator $l$ nearest to the first oscillator and chose $m$ again [if the $(l+m)$th oscillator had been permuted, the next value of $m$ was generated] and permuted the $l$th and $(l+m)$th oscillators: $\omega_l = (l+m-1)\Delta\omega/(N-1)$, $\omega_{l+m} = (l-1)\Delta\omega/(N-1)$, and so on. As a result, we obtained a system that consisted of the same elements but connected in the ''wrong'' order. In such a scheme, the frequency distribution is changed again from completely regular $(\Delta\omega^* = 0)$ to completely disordered $(\Delta\omega^* = 2\Delta\omega)$.

Omitting details of the distribution function, the difference between the three types of distribution can be presented

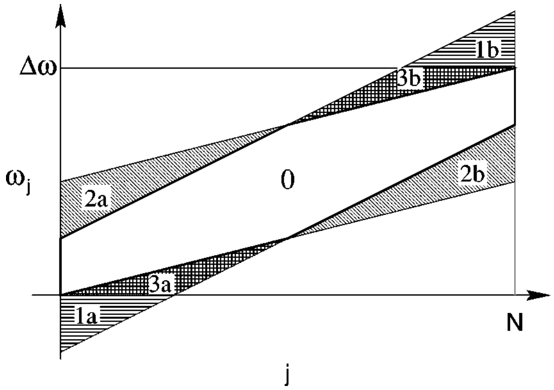


FIG. 6. Scheme of sets of frequencies for distributions of three different types at $\Delta\omega^*/\Delta\omega=1/2$ (see text).

schematically for $\Delta\omega^*/\Delta\omega=0.5$ (see Fig. 6 in which the boundaries of frequency ensembles $S_{I,II,III}$ are outlined). The three types of distributions have a common region 0. In the first case, it is supplemented by regions $1a,b$ and $3a,b$ ($S_I=S_0+S_1+S_3$), which expands the total frequency range. No additional expansion of frequency range occurs for a type II distribution. Moreover, an inverse (in a certain sense) process occurs—the frequencies from regions $2a,b$ that are concentrated in the middle of the frequency range ($S_{II}=S_0+S_2$) are added. For type III distribution, in contrast, more oscillators with frequencies at the edge of the range ($S_{III}=S_0+S_3$) are added. With these features taken into account, qualitative considerations lead us to the conclusion that the strongest effect should be expected for the case of a type II distribution when introduction of disorder is accompanied by a higher concentration of frequencies near the middle of the range, whereas for a type III distribution the effect is weaker due to the inverse effect produced by expansion of the frequency range. This tendency is observed in numerical experiments too (examples are given in Fig. 7). It is interesting that, even in the case of an absolutely random frequency distribution (at $\Delta\omega^*/\Delta\omega=1$ for type II distribution and at $\Delta\omega^*/\Delta\omega=2$ for type III distribution), oscillator death may be much weaker than for a monotonic frequency variation along the array.

## IV. MECHANISMS OF DISORDER INFLUENCE ON OSCILLATOR DEATH

A study of controlling the cluster pattern formation in the system under consideration [17] indicates that there exist at least two mechanisms of action of introduced disorder on oscillator death. One of them is involved with transformation of attractor (or attractors) as a whole, and the second, which is effective at multistability, implies that the attractors change only slightly, and only their attraction basins are transformed [17]. However, it was found that bistability regions occupy only a small portion of parameter space. So the role of the second mechanism is insignificant, if any. In our case, it could manifest itself only at $\Delta\omega\approx1$, i.e., at relatively weak oscillator death. However, the difference in the values of ''incoherent'' energy $\varepsilon(\Delta\omega^*=0)$ for four-cluster (0.630) and five-cluster (0.628) structures is small and comparable with the changes of their energies under the action of weak disorder without change in the number of clusters. In particular, for $\Delta\omega^*=0.035$, the values of $\varepsilon(\Delta\omega^*=0.035)$ are equal to 0.616 and 0.638, respectively.

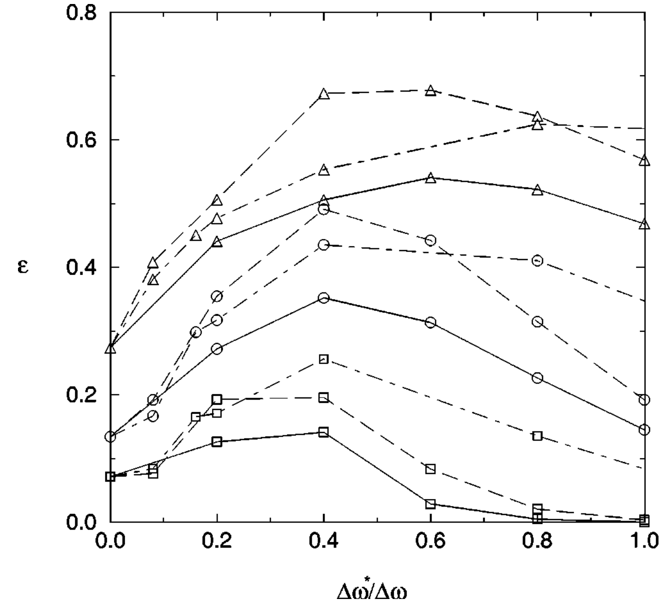


FIG. 7. Energy $\varepsilon$ vs disorder level for different types of distribution: I (solid lines), II (dashed lines), and III (dot-dashed lines). Again, the data represent, three different values of linear trend gradient $\Delta\omega/(N-1)$: relatively small, $\Delta\omega=3.0$ (△); moderate, $\Delta\omega=6.0$ (○, bold lines); and relatively large, $\Delta\omega=12.0$ (□).

The action of the first mechanism involved with transformation of the entire cluster structure is demonstrated in Figs. 3 and 4. It is clear from these figures that favorable situations are possible when disorder gives rise to synchronized clus-

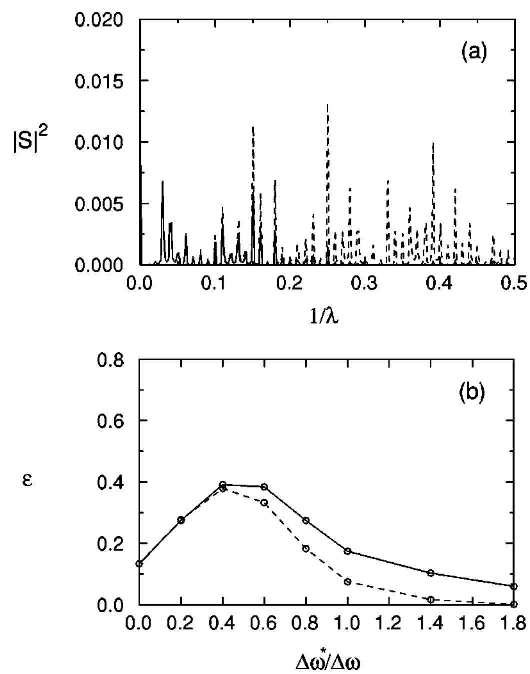


FIG. 8. (a) Power spectrum of random component of $\omega_j$ depicted in Fig. 3 (dashed line) and of filtered random component (solid line); (b) $\varepsilon$ vs relative disorder level; the same set of random numbers $\{\xi_j\}_{j=1}^N$ was used with different $\Delta\omega^*$. Solid line corresponds to filtered disorder, dashed line to nonfiltered one; $\Delta\omega=6.0$.

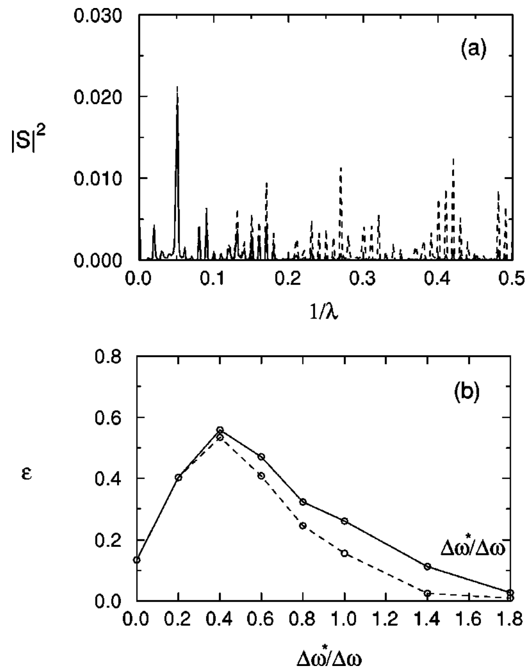


FIG. 9. The same as in Fig. 8, but for $\omega_j$ depicted in Fig. 4.

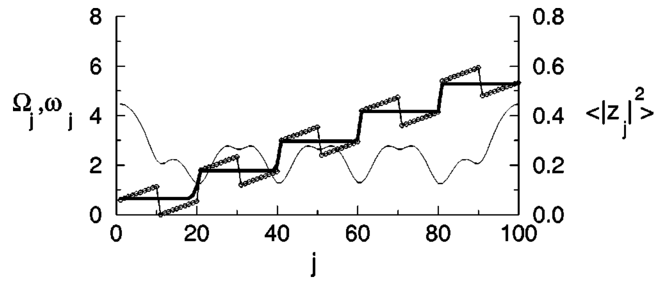


FIG. 10. The chain with meander of magnitude $\Delta\omega^*$ imposed on linear frequency trend, $\Delta\omega=6.0$ and $\Delta\omega^*=0.2\Delta\omega$. Time-averaged intensity profile $\langle|z_j|^2\rangle$ (solid line), averaged frequencies of oscillations $\Omega_j$ (bold line), and natural frequencies $\omega_j$ (circles connected by dashed line) are presented.

ters with oscillation intensities comparable to those observed in arrays without oscillator death. Formation of such clusters is caused by long wave components in the frequency distribution. Let us explain this on a very simple example, when superimposed frequency scatter has a purely sinusoidal dependence, so that

$$\omega_j=\Delta\omega\frac{j-1}{N-1}+\frac{\Delta\omega^*}{2}\sin\frac{2\pi}{N_\lambda}(j-1),$$

where $N_\lambda$ is the spatial period. Apparently, the as-formed frequency distribution at sufficiently large amplitude of deviations $(\Delta\omega^*/\Delta\omega\geqslant\pi/N_\lambda)$ will have nearly horizontal plateaus, so that $\omega_{j+1}-\omega_j\ll\Delta\omega/(N-1)$. For $N_\lambda\gg1$, the size of these plateaus will be large enough for the synchronized clusters formed under conditions of small mismatch to depend relatively weakly on the desynchronization action of the elements located in regions with large gradients of $\omega_j$.

This is confirmed by comparing the dependences $\varepsilon(\Delta\omega^*)$ obtained for disorder with filtered short wave components and unfiltered disorder. For the specific series of disorder corresponding to Figs. 3 and 4, the spectra and relevant dependences $\varepsilon(\Delta\omega^*)$ are given in Figs. 8 and 9. The random component of $\omega_j$ was filtered with the aid of a linear low pass filter,

$$\omega_j=\Delta\omega\frac{j-1}{N-1}+\Delta\omega^*F[\xi_j],$$

where

$$F[\xi_j]=(\xi_{j-1}+\xi_j+\xi_{j+1})/3$$

acts as a filter of high harmonics [22]. It is worthy of interest that the short wave components of spatial disorder weakly affect the value of energy $\varepsilon$. Therefore, it is actually demanded that the frequency distribution should have sufficiently extended plateaus ''on the average'' as is illustrated in Fig. 10. This figure presents the case of a distribution in which the frequency gradients of the initial distribution are retained everywhere except at several points. Jumps at some points provide nonmonotonic variation of frequency and conservation of the total range of frequency scatter as in the case of their monotonic variation. With the mechanism described above taken into account, the optimal value of frequency scatter $\Delta\omega^*_{opt}$ in Figs. 5, 7, 8, and 9 becomes clear. It is due to the fact that only a long wave component of sufficiently large amplitude can compensate the initial frequency gradient in the array.

## V. CONCLUSION

In this paper we demonstrated that introduction of random deviations from a regular, smooth trend of natural frequencies of oscillators of a one-dimensional oscillatory array can significantly decrease the degree of oscillator death in the system and thus increase the total intensity of oscillations. There exists an optimal level of disorder at which the oscillation energy of the system is maximized. The dependence of oscillation energy on the level of disorder (when the oscillator death is well pronounced in the absence of disorder) is bell shaped.

There are many examples of nontrivial, ''positive'' action of temporal or/and spatial irregularities on system dynamics. As a well-known example for the case of temporal irregularity we mention stochastic resonance [23]. Several examples for the case of spatial irregularity were already presented in the Introduction [4–6]. The effect described in this paper can be considered as another example of the nontrivial action of disorder in arrays of coupled oscillators.

## ACKNOWLEDGMENTS

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