# Teaching & Testing Mathematics Reading

**Carl C. Cowen**

**IUPUI**

**(Indiana University Purdue University Indianapolis)**

**M & T Seminar, 23 February 2010**

What are our goals for our students in our classes??

From the Audience:

General idea of [business calculus] is how quantities change and applications to their major.

Thinking (logic, etc) skills about problem solving

Not to be intimidated

Communicate: discussion and writing

Generate interest (not just in the grade)

Develop good academic habits and skills

What resources do we think our students have to achieve these goals?

From the Audience:

Class, Office Hours, Math Assistance Center

Textbook ($\Leftarrow\Leftarrow$!!)

Outside motivation (e.g. course required for major)

Classmates (How can we enhance the positive effects?)

Virtual connections

Internet resources $\cdots$

I want to argue that this goal:

*Our students should learn to read and,*

*thereby, understand mathematics.*

should be a conscious part of our teaching effort!

And to do this, we must test their ability to do so!

We are not only teaching them for the next test, or even for the next course,

we are teaching them for a lifetime of working and learning!

Very few students will ever have to give proof of a theorem (!)

Many *will* have to read and understand mathematical writing in their jobs.

Also, learning to read and understand mathematics will make learning to prove theorems more possible!

I do not believe that our current goals should be dropped, they are an important part of every mathematics course.

On the other hand, I do not believe that learning to read can be left simply to chance; that's what we've been doing and it hasn't worked!

To help them learn to read, we must give them opportunities to practice.

- discuss the meaning of a new theorem before proving it
- discuss hypotheses: many students do not know the role of a hypothesis
- ask them to restate the conclusion in a particular instance
- draw out corollaries that have not yet been stated

**Theorem.** *If $A$ is an $n$ by $n$ matrix and $\mu$ is a number such that*

$$\|A\| < |\mu|$$

*then $\mu I - A$ is invertible.*

Comment: This theorem is true for any submultiplicative norm on matrices; the norm used below is the one–norm:

$$\|A\| = \max\{\sum_{i=1}^{n} |a_{ij}| : j = 1, \cdots, n\}$$

What is the one–norm of the matrix

$$A = \begin{pmatrix} .7 & -.2 \\ .5 & 1.4 \end{pmatrix}$$

**Theorem.** *If $A$ is an $n$ by $n$ matrix and $\mu$ is a number such that*

$$\|A\| < |\mu|$$

*then $\mu I - A$ is invertible.*

$$\text{For} \quad A = \begin{pmatrix} .7 & -.2 \\ .5 & 1.4 \end{pmatrix} \quad \text{we have} \quad \|A\| = 1.6$$

What does this theorem say about the matrix

$$2I - A = \begin{pmatrix} 1.3 & .2 \\ -.5 & .6 \end{pmatrix}$$

**Theorem.** *If $A$ is an $n$ by $n$ matrix and $\mu$ is a number such that*

$$\|A\| < |\mu|$$

*then $\mu I - A$ is invertible.*

$$\text{For} \quad A = \begin{pmatrix} .7 & -.2 \\ .5 & 1.4 \end{pmatrix} \quad \text{we have} \quad \|A\| = 1.6$$

What does this theorem say about the matrix

$$I - A = \begin{pmatrix} .3 & .2 \\ -.5 & -.4 \end{pmatrix}$$

**Theorem.** *If $A$ is an $n$ by $n$ matrix and $\mu$ is a number such that*

$$\|A\| < |\mu|$$

*then $\mu I - A$ is invertible.*

$$\text{For} \quad A = \begin{pmatrix} .7 & -.2 \\ .5 & 1.4 \end{pmatrix} \quad \text{we have} \quad \|A\| = 1.6$$

Who can remind us of the definition of eigenvalue and tell some things about eigenvalues?

What does this theorem say about the eigenvalues of $A$?

# POP QUIZ

**Theorem.** *If $m$ and $n$ are positive integers such that*

$$2m = n^2 + 1$$

*then $m$ is the sum of the squares of two integers.*

**Proof.** Since $n^2 + 1$ is even, $n^2$ and therefore $n$ must be odd. That is, there is an integer $k$ so that $n = 2k + 1$. This means that

$$2m = n^2 + 1 = (2k+1)^2 + 1 = (4k^2 + 4k + 1) + 1 = 4k^2 + 4k + 2$$

It follows that

$$m = 2k^2 + 2k + 1 = k^2 + (k^2 + 2k + 1) = k^2 + (k+1)^2$$

which expresses $m$ as the sum of the squares of two integers. ∎

**Problem:**

$$2 \cdot 10805 = 21610 = 147^2 + 1$$

Write 10805 as the sum of the squares of two integers.

# POP QUIZ (page 2)

**Theorem.** *If $A$ is a diagonalizable matrix all of whose eigenvalues are non–negative, then there is a matrix $B$ with non–negative eigenvalues such that $B^2 = A$.*

**Proof.** Since $A$ is diagonalizable, there is an invertible matrix $S$ such that $L = S^{-1}AS$ is diagonal. The diagonal entries $\lambda_1, \lambda_2, \cdots, \lambda_n$ of $L$ (which are the eigenvalues of $A$) are non–negative by hypothesis.

$$\vdots$$

$$\vdots$$

$\blacksquare$

**Problem:** The matrix $A = \begin{pmatrix} 10 & -9 \\ 6 & -5 \end{pmatrix}$ has eigenvalues 1 and 4.

Find a matrix $B$ with positive eigenvalues such that $B^2 = A$.

Many thanks to Dick Hunt, J. J. Price, and Bob Zink for their support and these former colleagues and an anonymous referee for suggestions on these ideas.

Over the past decade and a half, there have been several sessions at national meetings at which faculty have presented ideas about how to get students to read. Some of their ideas:

- Give a reading assignment of section(s) from the textbook and give homework or a quiz on the material without discussing it in class beforehand (or ever).
- Ask students to read the section for each class before they come to class and by (say) one hour before class email the instructor with a question or statement that requires their having read the section already; include this in grade for course.
- Ask students to read the section for each class before they come to class and on random days, give students a quiz that would require them to have read the section.

IDEAS?

From the Audience:

Does this change their notions of authority? That is, are they less likely to accept the book/teacher/website without critical thinking?

Even more popular than worrying about reading mathematics, is getting students to learn to write mathematics!

The article this talk is based on (*Amer. Math. Monthly* **98**(1991)50–53.), as well as the slides for this presentation, are on my website:

`http://www.math.iupui.edu/~ccowen/Downloads.html`