Use of Technology in Teaching the Mathematics of Linear Algebra

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How can you explain this to your colleagues who don't use technology??

NONSENSE!!   We *all* use technology!!   We must *choose* which to use.

Since my first years of teaching, my *favorite* technology has been chalk on a slate blackboard!

Introduction of chalk and blackboard technology in 1801 revolutionized teaching!

However, this technology has limitations – and it is about the use of computer technology, as an add-on to the more traditional technology, that I want to speak today.

I use computer technology (usually MATLAB) to do a better job of:

- Responding spontaneously to student questions and suggestions $\cdots$
- Modeling recovery from errors for my students $\cdots$
- Illustrating interplay between ideas w/o computational impediments $\cdots$
- Motivating development of the mathematics and mastery of the ideas $\cdots$

In the old days, $\cdots$

(*carefully prepared answers, with correct arithmetic, for examples and homework problems*)

This means, at least as far as calculations go, I was stuck to the script!

Now, if students ask "What if?" questions, we can pursue them easily because we can create examples quickly and the machine can do the calculations effortlessly.

Perhaps more importantly, this spontaneity allows for natural reaction to student errors.

Now, $\cdots$

we can pursue a suggestion (*any suggestion!*)

are we done? (*Exit!*)

are we stuck?

reflect on what went wrong,

develop a new suggestion,

Repeat!

In addition to errors being a problem,

I believe students' ideas about errors interfere with their learning!

Students bring to our classes ideas about themselves and about us

– as to our expertise in mathematics.

Often, students believe that someone who knows mathematics well (like us) sees how to answer every question immediately and never makes conceptual or strategic errors in solving problems.

In particular, they sometimes infer that they are 'no good' at mathematics because they do not see how to answer every question immediately.

We *know* we do *not* always see the way to the solution of every problem and that we make errors – but we know how to recover from them!

When we use the strategy, $\cdots$

pursue a suggestion

are we done? (*Exit!*)

are we stuck?

reflect on what went wrong,

develop a new suggestion,

We are modeling how to recover from errors!!

And, the technology has made this easier to do!

Use of the computer technology allows me to bring in the theory more directly.

One example is in solution of systems of equations

– the first part of the course is about this problem.

It is easy to get students to learn elimination, but harder to get students to learn the theorems about solutions of systems.

Consider the system: $\begin{cases} 2a - b + c = 4 \\ -2a + 2b - c + 2d = -4 \\ -a - c + d = -3 \end{cases}$

```
>> A = [ 2  -1  1  0; -2  2  -1  2; -1  0  -1  1  ]

A =

     2    -1     1     0
    -2     2    -1     2
    -1     0    -1     1


>> A\[4 -4 -3]'

ans = [   2.5000;      1.0000;      0;    -0.5000   ]
```

```
>> null(A)

ans =

   -0.5477

   -0.3651

    0.7303

    0.1826
```

So the conclusion is that there are infinitely many solutions, and

$$
\begin{aligned}
a &= 2.5 - .5477t \\
b &= 1 - .3651t \\
c &= .7303t \\
d &= -.5 + .1826t
\end{aligned}
$$

Exercise: L.A. class assignment: Find the nullspace of the 4×5 matrix B.

John's answer was

$$\mathcal{N}(B) = \text{ span}\left\{ \begin{pmatrix} -2 \\ -2 \\ 0 \\ 2 \\ -6 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \\ 4 \\ -3 \\ 11 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \\ 2 \\ -4 \\ 13 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ -2 \\ 1 \\ -4 \end{pmatrix} \right\}$$

Mary's answer was

$$\mathcal{N}(B) = \text{ span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 4 \\ -1 \\ 5 \end{pmatrix} \right\}$$

Are their answers consistent with each other? (Explain your answer!)

One strategy is to find out whether each vector in John's set is in Mary's subspace and whether each vector in Mary's set is in John's subspace.

This is not difficult, requiring only the solution of 7 linear systems, or if reformulated as matrix equations, to solve two matrix equations. This is fairly easy to do by machine, but to do that by hand is quite tedious. (– *and I do this, at least partially, for them before proceeding!*)

Even so, there is a more efficient, although more sophisticated, strategy if you have a tool like MATLAB!

```
>> J=[-2 1 3 0; -2 5 5 -2; 0 4 2 -2; 2 -3 -4 1; -6 11 13 -4]

J =

    -2     1     3     0

    -2     5     5    -2

     0     4     2    -2

     2    -3    -4     1

    -6    11    13    -4


>> rank(J)

                              ans = 2
```

```
>> M = [1 -2 -1; 1 0 3; 0 2 4; -1 1 -1; 3 -2 5]

M =

     1    -2    -1

     1     0     3

     0     2     4

    -1     1    -1

     3    -2     5

>> rank(M)

                                        ans =  2


>> rank([J M])

                                        ans =   2
```

Finally, I use the technology to motivate the mathematics and to prepare the students for upcoming developments.

In the syllabus, I announce that one of the goals of the course is to enable the students to read the documentation for linear algebra software packages.

I do not usually have to tell the students about the commands `null` or `orth` in MATLAB: they come to expect that MATLAB has commands for everything, and someone discovers `null` and spreads the word before I tell them about it!

On the other hand, they don't like the answers!

For example, if C is the matrix

$$\begin{pmatrix} -2 & 1 & 3 & 0 \\ -2 & 5 & 5 & -2 \\ 0 & 4 & 2 & -2 \\ 2 & -3 & -4 & 1 \end{pmatrix}$$

By hand, they might find a basis for $\mathcal{N}(C)$ to be $u = (3, 0, 2, 2)$ and $v = (5, -2, 4, 0)$, a *nice* answer.

But using the machine,

```
>> null(C)


ans =

    0.7558    0.1482
   -0.2703    0.4637
    0.5940   -0.0558
    0.0533    0.8717
```

an *ugly* answer! Why does MATLAB always give *ugly* answers??

To summarize, I believe the technology helps me teach more effectively:

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`http://www.math.iupui.edu/~ccowen/Downloads.html`