Math 444 Recursive Sequence Problem

Define the function s from $\mathbb{N}$, the set of natural numbers, to $\mathbb{Q}$ in the following recursive (inductive) way:

$$s(1) = 2 \text{ and for } n \in \mathbb{N}, \quad s(n+1) = \frac{s(n)}{2} + \frac{1}{s(n)}$$

Prove: For each n in $\mathbb{N}$, we have $1 \leq s(n) \leq 2$.

Usually, we do not think about functions defined on the natural numbers in this way, instead, we call such a function a *sequence* and instead of writing $s(n)$ we write a_n for the terms of the sequence (actually values of the function) where $a_n = s(n)$. The following is the more usual way to state the same problem:

Definition We define a sequence $\{a_n\}_{n=1}^{\infty}$ by

$$a_1 = 2$$

and for each positive integer n,

$$a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}$$

Prove: For each n in $\mathbb{N}$, we have $1 \leq a_n \leq 2$.