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Math 164 (Cowen) **Test 1 (Practice)** **3 September 2007**

There are 5 pages and 20 questions. No partial credit! Scoring will be '110' for all correct, '100' for one incorrect, '90' for 2 incorrect, '80' for 3 incorrect, etc., to '−90' for all incorrect.

You will have 1 hour to complete this test!

For each of the questions 1 – 8, find the derivative of the given function.

(10 points) **1.** $f(x) = 4x^5 + 3\sqrt{x^{11}} - \frac{3}{\sqrt[4]{x}} - \frac{4}{x^8} = 4x^5 + 3x^{11/2} - 3x^{-1/4} - 4x^{-8}$

$f'(x) =$

$$20x^4 + \frac{33}{2}x^{9/2} + \frac{3}{4}x^{-5/4} + 32x^{-9}$$

(10 points) **2.** $g(t) = 3e^{4t} - 8.3\ln 5t$

$g'(t) =$

$$3e^{4t}(4) - 8.3\frac{1}{5t}(5)$$

(10 points) **3.** $y = 5.1\arcsin 2x - 3\arctan\frac{x}{5}$

$y' =$

$$5.1\frac{1}{\sqrt{1-(2x)^2}}(2) - 3\frac{1}{1+(\frac{x}{5})^2}\frac{1}{5}$$

(10 points) **4.** $h(w) = \dfrac{5}{\sqrt{16-w^2}} = 5(16-w^2)^{-1/2}$

$h'(w) =$

$$5\left(-\frac{1}{2}\right)(16-w^2)^{-3/2}(-2w)$$

(10 points) **5.** $r(\theta) = e^{\tan 5\theta}$

$r'(\theta) =$

$$e^{\tan 5\theta}(\sec 5\theta)^2(5)$$

(10 points) **6.** $f(t) = \ln(2 + e^{-3t^2})$

$f'(t) =$

$$\frac{1}{2 + e^{-3t^2}} e^{-3t^2}(-6t)$$

(10 points) **7.** $h(w) = \ln\left(\frac{5w^3 + \cos w}{3 + e^{2w}}\right) = \ln\left(5w^3 + \cos w\right) - \ln\left(3 + e^{2w}\right)$

$h'(w) =$

$$\frac{1}{5w^3 + \cos w}(15w^2 - \sin w) - \frac{1}{3 + e^{2w}}(e^{2w}2)$$

(10 points) **8.** $y = (x^8 + 5)^5 e^{3x^4}$

$y' =$

$$5(x^8 + 5)^4(8x^7)e^{3x^4} + (x^8 + 5)^5 e^{3x^4}(12x^3)$$

For each of the questions 9 – 20, find an indefinite integral or the definite integral, as indicated.

(10 points) **9.** $\int (5-4z)^6 \, dz =$

$$-\frac{1}{28}(5-4z)^7 + C$$

(10 points) **10.** $\int (2y^2+3)^5 y \, dy =$

$$\frac{1}{24}(2y^2+3)^6 + C$$

(10 points) **11.** $\int \left(3e^{2x}+1\right)^5 e^{2x} \, dx =$

$$\frac{1}{36}\left(3e^{2x}+1\right)^6 + C$$

(10 points) **12.** $\int 4\sin 5t - 2(\sec 3t)^2 \, dt =$

$$-\frac{4}{5}\cos 5t - \frac{2}{3}\tan 3t + C$$

(10 points) **13.** $\displaystyle\int \frac{11x}{144+x^2}\,dx =$

$$\frac{11}{2}\ln\left(144+x^2\right)+C$$

(10 points) **14.** $\displaystyle\int \frac{3}{25+4x^2}\,dx =$

Substitute $2x = 5u$, so that $dx = \frac{5}{2}du$. This gives

$$\begin{aligned}\int \frac{3}{25+4x^2}\,dx &= \int \frac{3}{25+25u^2}\,\frac{5}{2}du = \frac{3}{25}\frac{5}{2}\int \frac{1}{1+u^2}\,du = \frac{3}{10}\arctan u + C \\ &= \frac{3}{10}\arctan\frac{2x}{5} + C\end{aligned}$$

(10 points) **15.** $\displaystyle\int (\sin 2y)e^{\cos 2y}\,dy =$

Substitute $u = \cos 2y$, so that $du = -2\sin 2y\,dy$. This gives

$$\int (\sin 2y)e^{\cos 2y}\,dy = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{\cos 2y} + C$$

(10 points) **16.** $\displaystyle\int \frac{1}{t\sqrt{t^2-1}}\,dt =$

$$\operatorname{arcsec} t + C$$

(10 points) **17.** $\int_{-1}^{1} \frac{1}{\sqrt{4-a^2}}\, da =$

Substitute $2u = a$, so that $2du = da$. When $a = -1$, $u = -\frac{1}{2}$ and when $a = 1$, $u = \frac{1}{2}$. This gives

$$\begin{aligned} \int_{-1}^{1} \frac{1}{\sqrt{4-a^2}}\, da &= 2\int_{-1/2}^{1/2} \frac{1}{\sqrt{4-4u^2}}\, du = 2\frac{1}{2}\int_{-1/2}^{1/2} \frac{1}{\sqrt{1-u^2}}\, du \\ &= \arcsin(u)\big|_{-1/2}^{1/2} = \arcsin(\frac{1}{2}) - \arcsin(-\frac{1}{2}) \end{aligned}$$

(10 points) **18.** $\int_{0}^{3} \frac{y}{\sqrt{100-4y^2}}\, dy = \int_{0}^{3} y\left(100-4y^2\right)^{-1/2}\, dy$

Substitute $u = 100 - 4y^2$, so that $du = -8y\, dy$. When $y = 0$, $u = 100$ and when $y = 3$, $u = 64$. This gives

$$\begin{aligned} \int_{0}^{3} \frac{y}{\sqrt{100-4y^2}}\, dy &= \int_{0}^{3} y\left(100-4y^2\right)^{-1/2}\, dy = -\frac{1}{8}\int_{100}^{64} u^{-1/2}\, du \\ &= -\frac{1}{8}(2)\, u^{1/2}\Big|_{100}^{64} = -\frac{1}{4}\sqrt{64} - (-\frac{1}{4}\sqrt{100}) = -\frac{1}{4}(8) + \frac{1}{4}(10) = \frac{1}{2} \end{aligned}$$

(10 points) **19.** $\int_{0}^{5} \frac{1}{4+z^2}\, dz =$

Substitute $2u = z$, so that $2du = dz$. When $z = 0$, $u = 0$ and when $z = 5$, $u = \frac{5}{2}$. This gives

$$\begin{aligned} \int_{0}^{5} \frac{1}{4+z^2}\, dz &= \int_{0}^{5/2} \frac{1}{4+4u^2}\, 2du = \frac{2}{4}\int_{0}^{5/2} \frac{1}{1+u^2}\, du \\ &= \frac{1}{2}\arctan u\big|_{0}^{5/2} = \frac{1}{2}\arctan(\frac{5}{2}) - \frac{1}{2}\arctan(0) = \frac{1}{2}\arctan(\frac{5}{2}) \end{aligned}$$

(10 points) **20.** $\int_{-\pi/2}^{\pi/2} \frac{\cos\theta}{3+\sin\theta}\, d\theta =$

Substitute $u = 3 + \sin\theta$, so that $du = \cos\theta\, d\theta$. When $\theta = -\pi/2$, $u = 2$ and when $\theta = \pi/2$, $u = 4$. This gives

$$\int_{-\pi/2}^{\pi/2} \frac{\cos\theta}{3+\sin\theta}\, d\theta = \int_{2}^{4} \frac{1}{u}\, du = \ln u\big|_{2}^{4} = \ln 4 - \ln 2 = \ln\frac{4}{2} = \ln 2$$