Math 53000: Functions of a Complex Variable I (Class No: 10958)

Meets: TuTh 3:00 – 4:15p in LD 229
Final Exam: Thursday, May 8, 3:30p – 5:30p

Instructor: Carl Cowen **Office:** LD 224P **Phone:** 278-8846
Office Hours: TuTh 10:00-11:00, W 11:00-12:30, or by appointment
E-mail: `ccowen@math.iupui.edu`
URL: `http://www.math.iupui.edu/~ccowen/Math530.html`
Student Complex Analysis Seminar: Wednesdays, 9:00-10:25, LD 265

Complex Analysis, or the Theory of Functions of a Complex Variable, is a central topic in analysis at an advanced level. It is analogous to real analysis, but also quite different from real analysis, because complex differentiable functions are much more special than real differentiable functions. For this reason both real and complex analysis are foundations for further study in analysis. However, Complex Analysis is important in many other parts of mathematics as well, including algebraic geometry, number theory, and the geometry of manifolds. One of the main goals of the course is to make this distinction apparent to you, to help you develop an appreciation of the power of the function theoretic point of view, and to help you add it to your mathematical toolbox.

This is a graduate mathematics course, although (because it is actually a "dual level" course) advanced undergraduates will be welcome with permission of the instructor (email: `ccowen@math.iupui.edu`). This course is not at the research level, but it is expected that students taking this course are preparing for research in mathematics or a related discipline such as engineering or physics. Indeed, this course is one of the 'Core 4' subjects and this course should provide much of the background in preparation for the Qualifying Examination in Complex Analysis in the IUPUI mathematics PhD program. The grading policies (see below) have been chosen to reinforce these goals.

The necessary background for success in this course is one or two semesters of analysis (e.g. a graduate course in analysis or a course like IUPUI's Math 44400 and 44500).

TEXT: The official text for the course is *Complex Function Theory* (2^{nd} edition) by Donald Sarason, AMS, 2007 (ISBN: 978-0-8218-4428-1), about $40 (hardcover) from the AMS, less if you are a member. You will *not* be required to purchase any books as several books will be on reserve in the library and may be consulted there. No assignments will be made from any books. The material of the course will include topics from each of the chapters in Sarason's book, but not all of any one chapter.

The following books cover most of the material that is central, in my view, for this course, and any of them *could* have been chosen as the official text. The books marked with an asterisk(*) will be on reserve for this course in the IUPUI library.

Lars V. Ahlfors	*Complex Analysis**
Ruel V. Churchill	*Complex Variables and Applications**
John B. Conway	*Functions of One Complex Variable**
Steven G. Krantz	*A Guide to Complex Analysis*
Walter Rudin	*Real and Complex Analysis*
E. B. Saff and A. D. Snider	*Fundamentals of Complex Analysis with Applications* ···

Sarason's book and Krantz's book are usually cheaper than Ahlfors' or Conway's books. Churchill's and Saff and Snider's books more concerned with applications outside of mathematics than the others. Rudin's book integrates the development of real and complex analysis. Nevertheless, all of these and many others besides, are useful references for the course.

Topics

- The complex numbers: arithmetic, algebraic and geometric representations.
- Differentiation of complex valued functions of a complex variable
  - Cauchy-Riemann Equations
  - Conformality
  - Harmonic functions
- Linear fractional transformations
- Elementary functions: exponential, logarithmic, trigonometric
- Power series
- Integration of complex valued functions
- Cauchy's Theorem and Cauchy's integral formula
  - Mean Value Property, Liouville's Theorem, Schwarz's Lemma
  - Laurent series, Residue Theorem
- Local mapping, inverse mapping, open mapping theorems
- Preview: Conformal maps and the Riemann Mapping Theorem

Grading policy and philosophy

3 Keys to Learning Mathematics

1. Work lots of problems.
2. Memorize definitions and the statements of major theorems.
3. Work lots more problems.

Ordinarily, after mathematicians and scientists complete research on a topic, they write up their results and submit them for publication to an appropriate journal. To be accepted for publication, the work must be interesting, correct, and reasonably well written.

For the purposes of this class, all homework problems are, by hypothesis, interesting. Part of what you should be learning in this class, and others at this level, is how to write proofs that are both correct and well enough written to be understood by someone who does not know the result you are writing about. At this time, the international language of mathematics is English, so you will benefit by knowing how to communicate well in English. Therefore, your graded work will have comments about your writing and grammar in addition to comments about your mathematical ideas.

Weekly homework will be assigned and some of it will be identified when assigned to be collected and looked over. One problem from this list will be graded with more care and be permitted to be resubmitted, one time, after correction. There will be one quiz (January

21) and the grade on the quiz and the graded homework will constitute about 30% of your final grade. There will be two midterm tests counting about 20% each and a (cummulative) final exam on May 8 that will count about 30% also.

In addition to the regular lectures, there will be a **Student Complex Analysis Seminar** that meets each Wednesday, 9:00a-10:25a, in the Math Department's Seminar Room, LD 265. This will be a place for students to present solutions of homework exercises, or to ask questions about exercises, ask general questions not asked in class, or topics related to the class discussions but not central to the development of the subject. This seminar will be open to anyone, including students studying for the qualifying exams or needing to brush up on material previously studied. Problems to be collected the following day will not, directly, be part of the discussion, but related problems from the assignment are likely to be discussed.

Midterms and the Final Exam: The Midterms and the Final Exam will consist partly of questions concerning definitions, theorems, and examples from the course. You will *not* be asked to repeat proofs of the theorems from the course on the midterms or the exam. For example, "State Schwarz's Lemma and give an example of a polynomial of degree 2, not a multiple of z^2, that satisfies the hypotheses of the Lemma. Verify the conclusion of the Lemma for your example." would be a possible question on a test in this course, but "Prove Schwarz's Lemma." would not be. Problems from the assigned homework in this course, problems from the text or other reference books, or problems from the qualifying exam archive on the department website will be considered reasonable sources for questions on the midterms or the final exam. One of the goals of the midterms and the final exam will be to help you to become accustomed to the kind of exams that constitute the qualifying exams.

Campus Course Policies: IUPUI has certain policies that apply to every course; this course will follow these policies also. You should be familiar with the policies, especially those pertaining to academic integrity and adaptive services; they may be found at `http://registrar.iupui.edu/course_policies.html`
and `http://registrar.iupui.edu/withdrawal-policy.html`

Some Important Dates

Date	
January 14	First class
January 16	**No Official Class Meeting!**
January 21	**Quiz on complex numbers and arithmetic!**
March 10	Last day to withdraw with adviser's signature and automatic "W"
March 15-23	Spring Break!! no classes
April 7	Last day to withdraw with permission of adviser and instructor
May 1	Last class
May 8	Final Exam, 3:30–5:30p