NAME: KEY

Math 51100 (Cowen) **Quiz 1** **18 September 2017**

There are 2 questions, 2 pages, and 20 points on this quiz.

Show your work!! **Papers must be handed in by 4:20p**

1. Solve the following system. If there are no solutions, say so; if there is exactly one solution, say so and find the solution; if there are infinitely many solutions, find the general solution and give two solutions explicitly.

$$\begin{cases} a - b + c + d = 2 \\ -a + c + d = 1 \\ 2a - b + c = 3 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 2 \\ -1 & 0 & 1 & 1 & 1 \\ 2 & -1 & 1 & 0 & 3 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 2 \\ 0 & -1 & 2 & 2 & 3 \\ 0 & 1 & -1 & -2 & -1 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 1 & 0 & -1 & -1 & -1 \\ 0 & -1 & 2 & 2 & 3 \\ 0 & 0 & 1 & 0 & 2 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & 1 & 0 & 2 \end{array}\right)$$

5

$$\to \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 0 & 2 \end{array}\right)$$

$a = d + 1$ general solution

$b = 2d + 1$

$c = 2$

d arbitrary

2

$a = 1$

$b = 1$

$c = 2$

$d = 0$

2

$$\begin{pmatrix} 2 \\ 3 \\ 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 3 \\ 5 \\ 2 \\ 2 \end{pmatrix}$$

1

2. (a) Find the inverse of $A = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -2 & 1 \\ -1 & 2 & -2 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 & 1 & 0 \\ -1 & 2 & -2 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 2 & 3 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{array}\right)$$

$$\begin{pmatrix} 2 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & -1 & -1 \end{pmatrix}$$ this should be A^{-1}

7

(b) Is your answer in part (a) correct? Explain why or why not!!

To check $\begin{pmatrix} 1 & -1 & 2 \\ 1 & -2 & 1 \\ -1 & 2 & -2 \end{pmatrix} \begin{pmatrix} 2 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & -1 & -1 \end{pmatrix} =$

3

$$\begin{pmatrix} 2-1 & 2-2 & 3-1-2 \\ 2-2 & \cancel{2+1} & 3-2-1 \\ -2+2 & -2+2 & -3+2+2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

So answer above is correct!

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1. Solve the following system. If there are no solutions, say so; if there is exactly one solution, say so and find the solution; if there are infinitely many solutions, find the general solution and give two solutions explicitly.

$$\begin{cases} w + 2x + 2z = -2 \\ w + y - z = 1 \\ w - x + 2y - 2z = 4 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & -2 \\ 1 & 0 & 1 & -1 & 1 \\ 1 & -1 & 2 & -2 & 4 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 1 & -1 & 2 & -2 & 4 \\ 1 & 2 & 0 & 2 & -2 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & -1 & 1 & -1 & 3 \\ 0 & 2 & -1 & 3 & -3 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & -1 & 1 & -1 & 3 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right)$$

$$\to \left(\begin{array}{ccccc} 1 & 0 & 0 & -2 & -2 \\ 0 & -1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right) \quad 5$$

general solution

$w = 2z - 2$

$x = -2z$

$y = -z + 3$

2

infinitely many solutions

$z=0$	$z=1$	$z=2$	$z=3$
$y=3$	$y=2$	$y=1$	$y=0$
$x=0$	$x=-2$	$x=-4$	$x=-6$
$w=-2$	$w=0$	$w=2$	$w=4$
2	1		

2. (a) Find the inverse of $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & -1 & 0 \\ 0 & 1 & -2 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1. \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 1 \\ 0 & 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right)$$

So $A^{-1} = \begin{pmatrix} -2 & -3 & 1 \\ 2 & 2 & -1 \\ 1 & 1 & -1 \end{pmatrix}$

(b) Is your answer in part (a) correct? Explain why or why not!! 3

$$A A^{-1} = \begin{pmatrix} 1 & 2 & -1 \\ -1 & -1 & 0 \\ 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} -2 & -3 & 1 \\ 2 & 2 & -1 \\ 1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So our calculation of A^{-1} is correct

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1. Solve the following system. If there are no solutions, say so; if there is exactly one solution, say so and find the solution; if there are infinitely many solutions, find the general solution and give two solutions explicitly.

$$\left\{\begin{array}{rrrrcr} a & -b & +c & +d & = & -1 \\ -a & +2b & & -2d & = & 1 \\ a & +b & +2c & +d & = & 1 \end{array}\right.$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & 1 & -1 \\ -1 & 2 & 0 & -2 & 1 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -1 & 1 & 1 & -1 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 2 & 1 & 0 & 2 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -1 & 2 & 2 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 0 & 4 & 3 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & -1 & 2 & 2 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 1 & 0 & 0 & 4 & 3 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & -2 \end{array}\right)$$

general solution

$a = 3 - 4d$

$b = 2 - d$

$c = 2d - 2$

d arbitrary

$$\begin{pmatrix} 3 \\ 2 \\ -2 \\ 0 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -5 \\ 0 \\ 2 \\ 2 \end{pmatrix}$$

Some explicit solutions

2. (a) Find the inverse of $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 2 \\ 2 & -1 & 2 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ 2 & -1 & 2 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & -2 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -1 & 2 \\ 0 & 0 & 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & -2 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -1 & 2 \\ 0 & 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right)$$

So $A^{-1} = \begin{pmatrix} -2 & -1 & 2 \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

(b) Is your answer in part (a) correct? Explain why or why not!!

AA^{-1} should be I

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 2 \\ 2 & -1 & 2 \end{pmatrix}\begin{pmatrix} -2 & -1 & 2 \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} -2+2+1 & -1+1 & 2-1-1 \\ -2+2 & -1+2 & 2-2 \\ -4+2+2 & -2+2 & 4-1-2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So answer is correct

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$$\begin{cases} w + 2y + z = 2 \\ -w + x + y - z = -1 \\ 2w - x + z = 4 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 2 \\ -1 & 1 & 1 & -1 & -1 \\ 2 & -1 & 0 & 1 & 4 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 2 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & -1 & -4 & -1 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 2 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 \end{array}\right)$$

5

$$\to \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 4 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & 1 & -1 \end{array}\right)$$

General solution

$w = z + 4$

$x = 3z + 4$

$y = -z - 1$ $\quad z$ arbitrary

z

$$\begin{pmatrix} 6 \\ 10 \\ -3 \\ 2 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \\ -2 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ 1 \\ -2 \end{pmatrix}$$

2 1

So specific solutions

2. (a) Find the inverse of $A = \begin{pmatrix} 1 & -1 & 1 \\ -2 & 3 & 0 \\ 2 & -2 & 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ -2 & 3 & 0 & 0 & 1 & 0 \\ 2 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & 0 & -1 & -2 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -2 & 1 & 2 \\ 0 & 0 & -1 & -2 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 1 & 3 \\ 0 & 1 & 0 & -2 & 1 & 2 \\ 0 & 0 & -1 & -2 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 1 & 3 \\ 0 & 1 & 0 & -2 & 1 & 2 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right)$$

So inverse should be $\begin{pmatrix} -3 & 1 & 3 \\ -2 & 1 & 2 \\ 2 & 0 & -1 \end{pmatrix}$ 7

(b) Is your answer in part (a) correct? Explain why or why not!!

Check by multiplying ~~the~~ matrices

3

$$\begin{pmatrix} -3 & 1 & 3 \\ -2 & 1 & 2 \\ 2 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ -2 & 3 & 0 \\ 2 & -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -3-2+6 & 3+3-6 & -3+3 \\ -2-2+4 & 2+3-4 & -3+3 \\ 2-2 & -2+2 & 2-1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So answer is correct, product is I