Math 300 **Due November 1**

Homework 9

1. Using the terminology and ideas of Homework 8, discover, state, and prove a theorem that tells when a number can be written as the sum of three consecutive integers.

2. For each of the following, using your theorem from Exercise 1, decide which (if any) of the following numbers can be written as the sum of three consecutive integers. If the number can be written in this way, find the three integers. If the number cannot be written as the sum of three consecutive integers, show that it cannot.

(a) $n = 1346$

(b) $n = 3471$

(c) $n = 2761$

3. Discover, state, and prove a theorem that tells when a number can be written as the sum of six consecutive integers.

4. For each of the following, using your theorem from Exercise 3, decide which (if any) of the following numbers can be written as the sum of six consecutive integers. If the number can be written in this way, find the six numbers. If the number cannot be written as the sum of six consecutive integers, show that it cannot.

(a) $n = 2345$

(b) $n = 3471$

(c) $n = 2676$

5. Suppose f, g, and h are polynomials such that $f(x) = g(x)h(x)$. Prove that if α is a root of f, then α is a root of g or α is a root of h.

6. Suppose $f = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x$ is a polynomial with integer coefficients and $a_1 \neq 0$. Show that if α is a non-zero rational number that is a root of f with $\alpha = p/q$ for p and q integers, $q > 0$, and the greatest common divisor of p and q is 1, then q divides a_n and p divides a_1. (Note: You may use the result of Exercise 4 of Homework 6 if you wish.)

7. Find the multiple roots of $x^6 - 17x^4 + 88x^2 - 144$ and the multiplicity of each.