Math 300 **Due October 25**

Homework 7

1. The polynomial $x^2 - 5x + 6$ has roots 2 and 3. Find the roots of $6x^2 - 5x + 1$.

2. Let $f(x) = 2x^4 - 3x^2 + 4x - 5$ and let $g(x) = x^2 - 5x - 2$. Find $q(x)$ and $r(x)$ so that the degree of $r(x)$ is less than the degree of $g(x)$ and

$$f(x) = g(x)q(x) + r(x)$$

3. Use the Euclidean algorithm to find the greatest common divisor of the following pairs of polynomials:

(a)	$x^2 + 24x + 63$	and	$x^3 - 37x - 84$
(b)	$x^3 - 37x - 84$	and	$x - 7$
(c)	$x^4 + 118x - 35$	and	$x^2 - 2x - 35$

4. What are the common roots of the pairs of polynomials (note the overlap with Exercise 3):

(a)	$x^2 + 24x + 63$	and	$x^3 - 37x - 84$
(b)	$x^4 + 118x - 35$	and	$x^2 - 2x - 35$
(c)	$x^4 - x^3 - 7x^2 + x + 6$	and	$2x^4 - 7x^3 - 20x^2 + 49x + 60$

5. Suppose

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots a_2 x^2 + a_1 x + a_0$$

is a polynomial with $a_n \neq 0$ and $a_0 \neq 0$. Show that if α is a root of f, then $1/\alpha$ is a root of the polynomial

$$g(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots a_{n-2}x^2 + a_{n-1}x + a_n$$

6. (a) The polynomial $x^3 - 37x + 84$ has roots -7, 3, and 4. What are the roots of $84x^3 - 37x^2 + 1$?

(b) The polynomial $2x^3 - x^2 - 16x + 15$ has roots -3, $5/2$, and 1. What are the roots of $15x^3 - 16x^2 - x + 2$?