Math 300 **Due October 9**

Homework 4

1. (*Casting out nines*)
Suppose an integer n has decimal representation $n = d_k d_{k-1} \cdots d_2 d_1 d_0$. The sum of the digits of n is $s = d_k + d_{k-1} + \cdots + d_2 + d_1 + d_0$. Prove that n is divisible by 9 if and only if s, the sum of the digits of n, is divisible by 9. (For example, we can tell that $37,215$ is divisible by 9 because $3 + 7 + 2 + 1 + 5 = 18$ is divisible by 9.)

2. Suppose m, n, and d are positive integers. Prove that the remainder when dividing m by d is equal to the remainder when dividing n by d if and only if $m - n$ is divisible by d.

3. (More on *Casting out nines*) Use the ideas of Exercises 1 and 2 to show the following. The notation in this exercise is the same as in Exercise 1.

(a) The remainder when dividing n by 9 is the same as the remainder when dividing s, the sum of the digits of n, by 9.

(b) Prove that n is divisible by 3 if and only if s, the sum of the digits of n, is divisible by 3.

4. Use Theorem 11 of Chapter 1 of *Discourses on Algebra* to find all the rational roots of these polynomials:

(a)	$x^2 - 24x + 63$	(b)	$x^3 - 37x - 84$
(c)	$x^3 - 42x - 49$	(d)	$x^4 - 118x - 35$