Math 300 **Due October 4**

Homework 3

1. Use the Eucilidean Algorithm to find the greatest common divisor of each of given pairs of integers (Note that the first two of these are the same as 5(c) and 5(d) on Homework 2):

(a)	3003 and 2805	(b)	11433 and 23051
(c)	7469 and 2464	(d)	2689 and 4001
(e)	2947 and 3997	(f)	1109 and 4999

2. For each pair of numbers m and n in Exercise 1, find integers x and y so that the greatest common divisor d of m and n satisfies $d = mx + ny$.

3. Let a, b, and m be non-zero integers and suppose d is the greatest common divisor of a and b. Prove that md is the greatest common divisor of ma and mb.

Definition If b and c are integers, not 0, such that $b|m$ and $c|m$, we say *m is a common multiple of b and c.* Of course, bc is a common multiple of the integers b and c and, indeed, if n is any integer, nbc is also a common multiple of b and c. Thus, any pair of non-zero integers has infinitely many positive common multiples. The *least common multiple of b and c* is the smallest of the positive common multiples of b and c.

Reading: Let b and c be positive integers and let d be the greatest common divisor of b and c. This means that there are integers p and q so that $b = dp$ and $c = dq$. Now the greatest common divisor of p and q is 1. If that were not the case, say $p = rp'$ and $q = rq'$ for some integer $r > 1$, then $b = drp'$ and $c = drq'$ and dr would be a common divisor of b and c that is larger than d, the greatest common divisor, which is impossible.

Now $m = dpq$ is a multiple of b, because $m = bq$, and m is a multiple of c, because $c = mp$. That is, m is a common multiple of b and c. On the other hand, if m' is a positive multiple of b, say $m' = bs = dps$, and m' is a positive multiple of c, say $m' = ct = dqt$, then $\frac{m'}{d} = ps = qt$. Since p and q have no factors in common, the fact that $q|(ps)$ means $q|s$ and the fact that $p|(qt)$ means $p|t$. Thus, $m|m'$ and $m \leq m'$. Since this is true for all positive multiples, m', we see that m is the least common multiple of b and c.

4. Find the least common multiple of each of given pairs of integers:

(a)	24 and 84	(b)	525 and 315
(c)	3003 and 2805	(d)	11433 and 23051

5. Let a and b be non-zero integers and let d be the greatest common divisor of a and b and let m be the least common multiple of a and b. Prove that $md = |ab|$.