Math 300 **Due October 2**

Homework 2

Definition If a and b are integers, $a \neq 0$, we say b *is divisible by* a or a *divides* b, and write $a|b$, if there is an integer x so that $b = ax$.

1. In the following statements, suppose a, b, c, x, and y are integers.
(a) Show that if $a|b$, then $a|(bc)$.
(b) Prove that if $a|b$ and $b|c$, then $a|c$.
(c) Show: If $a|b$ and $a|c$, then $a|(bx + cy)$ for any integers x and y.
(d) Prove: If $a|b$ and $b|a$, then $a = \pm b$.

2. Use the fact that every integer is either even or it is odd to show that for all integers, n, the number $n^2 - n$ is divisible by 2.

3. Show that for each integer n, either $n - 1$ is divisible by 3 or n is divisible by 3 or $(n + 1)$ is divisible by 3.

4. (a) Show that for each integer n, the number $n^3 - n$ is divisible by 3.
(b) Prove that for each integer n, the number $n^3 - n$ is divisible by 6.

Definition If b and c are integers, not 0, such that $a|b$ and $a|c$, we say *a is a common divisor of b and c*. Of course, 1 is divisor every integer, so for any integers b and c, 1 is a common divisor of b and c. Since every positive divisor of b is less than or equal to $|b|$, there are only finitely many divisors of b, and every pair of integers has only finitely many common divisors. The *greatest common divisor of b and c* is the largest of the positive, common divisors of b and c.

For example, the common divisors of 63 and 147 are ± 1, ± 3, ± 7, and ± 21, so the greatest common divisor of 63 and 147 is 21.

5. Find the greatest common divisor of each of given pairs of integers:

(a)	24 and 84	(b)	525 and 315
(c)	3003 and 2805	(d)	11433 and 23051