Math 300 **For Discussion, December 11 and 13**

Homework 14

1. Let p, q, and r be statements. Use truth tables to show that the statement $\sim [p \rightarrow (q \wedge r)]$ and the statement $[p \wedge (\sim q)] \vee [p \wedge (\sim r)]$ are equivalent.

2. Let P be the truth set for the statement $p(x)$, Q the truth set for the statement $q(x)$, and R the truth set for the statement $r(x)$. What is the truth set of

$$r(x) \wedge (q(x) \vee (\sim p(x)))$$

3. What is the negation of the statement

$$(\exists x)[p(x) \rightarrow (q(x) \wedge r(x))]$$

4. For this question, the universal set for the variable n is the integers. Let $p(n)$ be the statement 'n is even', let $q(n)$ be the statement 'n is divisible by 3', and let $s(n)$ be the statement '$n^2 - n$ is divisible by 4'.

(a) Using the symbols of symbolic logic as in exercise 3 and the notation above, translate the following statement into symbols:

'*For all integers n, if n is even and not divisible by 3, then $n^2 - n$ is divisible by 4.*'

(b) Write the negation of the symbolic statement in (a) above.

(c) Translate into words the symbolic statement you have in (b).

(d) Either the statement in (a) or the statement in (c) is true. Decide which is true and show that it is.

5. (a) Suppose p and q are integers with $1 < p < q$. Assuming the greatest common divisor of p and q is 1, in the computation of the decimal expansion of $\frac{p}{q}$, what is the largest number of digits that can occur before the digits of the expansion begin the repeating pattern?

(b) For each of the rational numbers below, find the decimal expansion.

$$\frac{13}{40} \qquad \frac{6}{7} \qquad \frac{16}{21} \qquad \frac{3}{19}$$

6. For each of the repeating decimals below, express the rational number as a quotient of integers, reduced to lowest terms.

$$.31\overline{857142} \qquad 3.214\overline{4193} \qquad .\overline{047619}$$

7. Find the greatest common divisor of each of given pairs of integers:

$$(a) \quad 1575 \ \text{ and } \ 735 \qquad\qquad (b) \quad 3102 \ \text{ and } \ 2673$$

8. The integers 238 and 159 have greatest common divisor 1. Find integers x and y so that $238x + 159y = 1$.

9. Let p and q be non-zero integers and suppose d is the greatest common divisor of p and q.

(a) Prove that d^2 is the greatest common divisor of p^2 and q^2.

(b) Show that d^2 is always a common divisor of $2p^2$ and q^2.

(c) Is d^2 always the greatest common divisor of $2p^2$ and q^2? (Either prove that it is always true, or give an example to show that it is sometimes false.)

10. Find all the rational roots of these polynomials:

(a)	$x^3 - 39x - 70$	(b)	$x^4 + 118x - 35$
(c)	$2x^3 + x^2 - 18x - 9$	(d)	$12x^3 + 16x^2 - 7x - 6$

11. The integers -3, -2, -1, and 2 are roots of the polynomial

$$p(x) = x^6 + 8x^5 + 18x^4 - 8x^3 - 79x^2 - 96x - 36$$

(a) Which of these integers, if any, are multiple roots of p? Justify your answer!

(b) How many other real or complex numbers are roots of p?

12. Find a polynomial, p, of degree three or less, that satisfies $p(-1) = 1$, $p(2) = -3$, $p(4) = 1$, and $p(5) = -2$.

13. Prove that $k^2 - k + 2$ is not divisible by 3 for any integer k.

14. Prove by induction that the sum of the first n odd squares (that is, the squares of odd integers) is $\dfrac{n(2n-1)(2n+1)}{3}$

15. Let $a_1 = 2$ and let $a_{n+1} = a_n^2 + 2a_n$ for each positive integer n.

(a) Find the first five terms of the sequence $\{a_n\}$.

(b) Using induction, prove that $a_n \geq 2^n$ for all positive integers n.