Homework 11

Read Chapter 4, Section 4 "Proof by Mathematical Induction" in *Bridge to Mathematics*

1. Prove that the sum of the first n integers is $n(n+1)/2$.

2. Prove that the sum of the squares of the first n integers is $n(n+1)(2n+1)/6$.

3. Prove that the sum of the cubes of the first n integers is $n^2(n+1)^2/4$.

4. Find a formula for the sum of the first k even integers and show that your formula works.

5. Find a formula for the sum of the first k odd integers and show that your formula works.

6. The sequence a_n is defined recursively by $a_1 = 1$ and $a_{n+1} = 1 + \dfrac{1}{2 + a_n}$.

    (a) Write the first 4 terms of the sequence.

    (b) Show that for each positive integer n, we have $a_n \geq 1$.

    (c) Use your answer to part (b) to show that for each positive integer n, we have $a_n \leq 4/3$.