Math 300 **Due November 15**

Homework 10A

Definition A finite set of integers is said to be *a set of consecutive even integers* if there are integers m and j so that m is even and the given integers are m, $m+2$, $m+4$, $\cdots$, $m+2j$. Similarly, a finite set of integers is said to be *a set of consecutive odd integers* if there are integers m and j so that m is odd and the given integers are m, $m+2$, $m+4$, $\cdots$, $m+2j$.

For example, the set $\{17, 19, 21, 23, 25\}$ is said to be a set of five consecutive odd integers because the set consists of 5 integers and (using $m = 17$, an odd integer, and $j = 4$ in the definition) the integers are 17, $17+2$, $17+4$, $17+6$, and $17+8$.
Similarly, the set $\{14, 16, 18, 20\}$ is said to be a set of four consecutive even integers because the set consists of 4 integers and (using $m = 14$, an even integer, and $j = 3$ in the definition) the integers are 14, $14+2$, $14+4$, and $14+6$.

1. (a) Prove: If n is the sum of four consecutive even integers, then n is divisible by 4.

(b) Prove: If n is the sum of four consecutive odd integers, then n is divisible by 4.

(c) Write or try to write 12492 as the sum of four consecutive even integers. Write or try to write 12492 as the sum of four consecutive odd integers.

(d) Write or try to write 11504 as the sum of four consecutive even integers. Write or try to write 11504 as the sum of four consecutive odd integers.

(e) Figure out what is going on in part (c) and part (d). Explain what is going on by formulating and proving two theorems of the form "If an integer n $\cdots$, then n is the sum of four consecutive even integers." and "If an integer n $\cdots$, then n is the sum of four consecutive odd integers."

2. The polynomial $f(x) = x^7 - 6x^5 - 5x^4 + 9x^3 + 30x^2 - 45$ has two complex roots that are not real, and five (counting mutliplicity) real roots, none of which are rational. Find the multiple roots of f.

3. Let f be the polynomial $f(x) = x^7 - 6x^5 - 5x^4 + 9x^3 + 30x^2 - 45$, as in problem 2. The polynomial $g(x) = x^5 - 2x^3 - 5x^2 + 10$ has two complex roots that are not real, and three (counting mutliplicity) real roots, none of which are rational. Find the common roots of f and g.