

## Homework S5 (final version)

1. Do problem 2 on page 284 of the text.
2. (a) Find an expression for the Taylor polynomials of  $e^{-x}$ , estimate  $|E_n(x)|$ , and determine the sign of  $E_n(x)$ .  
 (b) Use part (a) to show that for  $k$  a positive integer,
 
$$T_{2k+1}(x) < e^{-x} < T_{2k}(x)$$
 where  $T_n(x)$  is the Taylor polynomial of degree  $n$  for  $e^{-x}$  for  $a = 0$ .  
 (c) Find an expression for the Taylor polynomials of  $e^{-t^2}$  and an inequality similar to that of part (b) for  $e^{-t^2}$  and the polynomials.  
 (d) Find an approximate value for  $e^{-1/4}$  that is correct to within  $.5 \times 10^{-4}$ , that is, to four place accuracy.

3. Do problem 7 on page 285 of the text.

4. Do problem 9 on page 285 of the text.

5. Use problem 2 above to find an approximate value for

$$\int_0^1 e^{-t^2} dt$$

with an error of less than .001.

On **separate pages** (they will be handed in separately) use *Maple* to do problems 1 – 5 again; specifically:

6. What is the title of Section 7.9, page 286?
7. Find the Taylor polynomial of degree 14 for  $\cos(x)$ .
8. (a) Find the Taylor polynomial of degree 14 for  $e^{-x}$ .  
 (c) Find the Taylor polynomial of degree 14 for  $e^{-t^2}$ .  
 (d1) Find an approximate value, expressed as a rational number, for  $e^{-1/4}$  that is correct to within  $.5 \times 10^{-4}$ .  
 (d2) Find an approximate value, expressed as a decimal number, for  $e^{-1/4}$  that is correct to within  $.5 \times 10^{-4}$ , that is, to four place accuracy.
9. Find an estimate for  $\int_0^{1/2} 1/(1+x^4) dx$  that satisfies  $.493948 < \text{answer} < .493958$ .

10. Use the first three non-zero terms of the Taylor series for  $\sin(x)$  to find an approximate value, expressed as a rational number, for  $\int_0^1 \sin(x)/x dx$ .
11. Use the ideas of problems 2 or 8 above to find an approximate value, expressed as a rational number and with an error of less than .001, for

$$\int_0^1 e^{-t^2} dt$$

Some Useful Tips:

- These notes are for use in the “Classic” *Maple* version: on a PC in an *IUPUI* lab, from the “Start” menu, you need to find *Maple* in the “departmentally sponsored” links and then choose “Maple Classic” from the list; on a Mac, start *Maple* and choose “TEXT” on the bar above the window that appears.
- Every *Maple* command must end with a “;” in order for it to be executed.
- To get help with a *Maple* command, enter `help([command]);` or `help(["phrase"]);` at the prompt. For example, `help(cos);` will tell you how to use the “cos” command and `help("partial derivative");` will give you a definition of the phrase “partial derivative” and point to relevant commands.
- The following commands might be helpful: `eval`, `evalf`, `taylor`, `factor`, `ifactor`, `simplify`, `factorial`, `ln`, `log`, `exp`, `sin`, `cos`, `tan`, `arcsin`, `arctan`, `convert`. You will probably want to use `help` to find out how to use some of them.
- In *Maple* the character `%` refers to the last calculation, and `%%` the one before that, so entering `120!`; at the prompt gives you 479001600 and then entering `ifactor(%);` at the next prompt gives you  $(2^{10})(3^5)(5^2)(7)(11)$