# Math 54600: Introduction to Functional Analysis (Class No: 31049)

**Meets:** MW 10:30 – 11:45p in LD 002
**Final Exam (optional):** Monday, December 16, 10:30a – 12:30p

**Instructor:** Carl Cowen **Office:** LD 224P **Phone:** 278-8846
**Office Hours:** M 12:00-1:00 and MW 4:00-5:15, or by appointment
**E-mail:** `ccowen@math.iupui.edu`
**URL:** `http://www.math.iupui.edu/~ccowen/Math546.html`

Functional analysis is a central topic in analysis at an advanced level and is at the foundation of many parts of analysis, including differential equations (both ordinary and partial), mathematical physics, numerical analysis, signal processing and control theory in engineering, and complex and real analysis, in addition to the subjects in the areas of Modern Analysis and Geometry that are primarily functional analysis: operator theory, $C^*$ and operator algebras, and non-commutative geometry, topology, and probability. Functional analysis extends the theory of linear algebra (over the real or complex fields) to infinite dimensional spaces and thereby requires the conscious addition of continuity to get problems and theorems applicable to a broad variety of important areas of applications and mathematics.

This is a graduate mathematics course, although (because it is actually a "dual level" course) advanced undergraduates will be welcome with permission of the instructor (email: `ccowen@math.iupui.edu`). This course is not at the research level, but it is expected that students taking this course are preparing for research in mathematics or a related mathematical discipline such as engineering or physics. This course would often provide part of the background for preparation for an Advanced Topics Examination in the IUPUI mathematics PhD program. The grading policies (see below) have been chosen to reinforce these goals.

The necessary background for success in this course is one or two semesters of analysis (e.g. a graduate course in analysis or IUPUI's Math 44400 and 44500) and an undergraduate course in linear algebra.

TEXT: The official text for the course is *A Guide to Functional Analysis* by Steven G. Krantz, MAA, 2013 (ISBN: 978-0-88385-357-3, about $25 (PDF), $50 (hardcover)). You will *not* be required to purchase any books as several books will be on reserve in the library and may be consulted there. No assignments will be made from any books. The material of the course is approximately Chapters 1, 2, 3, 4, 8, and 9 from Krantz's book with a little of Chapters 5 and 6 added.

The following books cover most of the material that is central, in my view, for this course, but they each have different perspectives than mine on learning the subject. Nevertheless, they are useful references for the course. The books marked with an asterisk(*) will be on reserve for this course in the IUPUI library.

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| J. B. Conway | *A Course of Functional Analysis** |
| J. B. Conway | *A Course in Operator Theory* |
| B. D. MacCluer | *Elementary Functional Analysis* |
| R. E. Megginson | *Introduction to Banach Space Theory** |
| A. Taylor and D. Lay | *Introduction to Functional Analysis* |
| P. R. Halmos | *Introduction to Hilbert Space* ···* |

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| P. R. Halmos | *Hilbert Space Problem Book** |
| W. Rudin | *Real and Complex Analysis** |
| W. Rudin | *Functional Analysis* |

The following books are less central, in my view, for the material of this course, but they might be useful references. Except for Baggett and Groetsch, they are more advanced than our text. None of these books is on reserve in the library.

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| L. W. Baggett | *Functional Analysis: A Primer* |
| S. K. Berberian | *Lectures in Functional Analysis* |
| C. W. Groetsch | *Elements of Applicable Functional Analysis* |
| Dunford and Schwartz | *Linear Operators* (3 Volumes) |
| Riesz and Sz. Nagy | *Functional Analysis* |
| K. Yosida | *Functional Analysis* |

## Topics

- Definitions, examples, and elementary properties
- Linear functionals and convexity: extension and separation theorems, weak topologies, Krein-Milman theorem
- Duality
- Completeness: uniform boundedness, open mapping, and closed graph theorems
- Hilbert spaces and generalized Fourier series
- Operators: spectrum, spectral mapping theorem, Riesz functional calculus
- Compact Operators
- Spectral theorem for self-adjoint and normal operators on Hilbert space
- Unbounded operators (if time permits)

## Grading policy and philosophy

**3 Keys to Learning Mathematics**

1. Work lots of problems.
2. Memorize definitions and the statements of major theorems.
3. Work lots more problems.

Ordinarily, after mathematicians and scientists complete research on a topic, they write up their results and submit them for publication to an appropriate journal. To be accepted for publication, the work must be interesting, correct, and reasonably well written. For the purposes of this class, all homework problems are, by hypothesis, interesting. After you submit them, they will be accepted if they are correct and reasonably well written. (Note that neither professional journals, your thesis committee, nor this class will accept work that is half correct.)

Publication of jointly authored research in mathematics is frequent and acceptable: the authors do the research together and write up a single account of their work for submission with all their names as authors.

There is an unwritten code of ethics governing joint publication. Whether work discussed with another mathematician is "joint" or not is determined by the individuals involved by mutual consent. One standard is that the work is jointly authored if the contribution of each author is "significant". It is unethical to submit jointly authored work as yours alone. It is equally unethical to include an author who has contributed nothing. Assignment of credit by professional communities for jointly authored work is always problematic and ranges from the generous: each author gets full credit for the whole paper; to the blantantly unfair: you get no credit, the other author gets full credit.

*Joint authorship of homework will be acceptable in this class* (to encourage discussion of the problems among you) and the usual ethical standards and the usual procedures apply: one manuscript with all authors names will be submitted for grading. Credit will be given for a problem only after the problem has been accepted as correct and reasonably well written. For a paper with $n > 1$ authors, credit to each author will be $2/n$, so, for example, for an individually worked problem, the author will get 1 credit; for a problem with two authors, *each* author will receive $2/2 = 1$ credit; whereas for a problem with three authors, *each* author will receive $2/3$ credit.

> Grades will be based on submitted homework (35%)
> and accepted homework (65%),
> with an *optional* written final examination given
> during finals week for extra credit (see below).

HOMEWORK: You will be given, sporadically, lists totaling more than 100 problems; you are encouraged to do as many of these as possible. The problems are of uneven difficulty: some are very easy; some of the more difficult ones are starred (*). Some of the homework problems will be of special interest to certain groups of students and will extend the work done in class, e.g. problems on unbounded differential operators, spaces of analytic functions, indefinite quadratic forms. You may turn in the problems in any order, and at any time, except that half the submitted problems must be on material from the first half of the course (on spaces, roughly speaking) and half on material from the second half of the course (on operators, roughly speaking).

- You should submit *at least* 30 problems during the semester:
  at least 4 by September 16,
  at least 4 more by September 30,
  at least 4 more by October 9,
  at least 4 more by October 28,
  at least 4 more by November 11,
  at least 4 more by November 25,
  *No solutions or corrections accepted after 5:00p, Friday, December 13.*

  Grades for the submission of homework will be the percentage of these goals.
- Rejected problems may *(should)* be resubmitted after correction.
- Grades for accepted homework will be computed as follows:

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| accepted problem credits $< 15$ | C |
| $15 \leq$ accepted problem credits $< 25$ | B |
| $25 \leq$ accepted problem credits | A |

FINAL EXAM: The Final Exam will consist entirely of questions concerning definitions and theorems from the course. There will be no "problems" on the final exam. The goal of the final exam is to encourage you to learn the facts of the course, and to provide a way to improve your grade, if you wish. An 'A' on the final exam will add 5 accepted problem credits to your accepted problems score.

**NOTE:** I expect everyone to get an "A" for the course!

CAMPUS COURSE POLICIES: IUPUI has certain policies that apply to every course; this course will follow these policies also. You should be familiar with the policies; they may be found at `http://registrar.iupui.edu/course_policies.html`