11/6/17, revised 12/31/17

Math 64600: Functional Analysis (Class No: 31635)

Meets: TuTh 9:00 – 10:15a in IT 065

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Office Hours: MW 10:00 – 11:30a, Tu 12:00-1:15p, or by appointment

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Canvas page: SP18-IN-MATH-64600-31635

Advanced Functional Analysis is a topics course in analysis at an advanced level and is a sequel to Intro to Functional Analysis, Math 54600. In Spring 2018, the semester will be divided into three fairly separate functional analysis areas, spending about a third of the time on each topical area.

We will begin with an introduction to Banach and C^*-algebras, the latter being an abstraction of the algebra of bounded operators on a Hilbert space. This can be seen as a continuation of the last part of Math 54600. At the beginning, we will redo the spectral theorem for self-adjoint operators from a more abstract point of view, in a way that takes advantage of the C^*-algebra structure. We will also develop the idea of the maximal ideal space, which can be identified as the set of (non-zero) multiplicative linear functionals on a Banach algebra.

In the second part of the course, we will continue the discussion of closed operators that were defined in Math 54600, but not studied much beyond proving that a closed operator defined on the whole of a Banach or Hilbert space is bounded. For example, differentiation is a linear transformation defined for some complex valued functions on the real line. We know from calculus, that if functions are defined on the unit interval and the norm is defined as the maximum value of the function, then differentiation is an unbounded operator. The main thrust of this part of the course will be to study unbounded operators defined on dense subspaces in Hilbert spaces to see how to expand the ideas from the study of bounded operators on Hilbert spaces and develop greater understanding of more general closed operators. The study of ordinary and partial differential equations leads to many examples of these operators, although this course will be focused on the general properties of these operators, not on the differential equations problems themselves.

Finally, we will study aspects of operator theory on Hilbert spaces of analytic functions that permit deeper understanding of a variety operators quite different from the operators arising in more classical studies of operators on Banach and Hilbert spaces. In particular, this part of the course will connect the function theory of analytic functions in one or more variables with functional analysis to show how ideas from functional analysis can lead to conclusions in function theory and vice versa. More specifically, we will introduce the Hardy, H^2, and Bergman, A^2, and other Hilbert spaces on the unit disc, polydisk, or unit ball in $\mathbb{C}^n$, and consider special operators on them. In particular, we will include Toeplitz and composition operators which provide a broader collection of operators than usually arise in the Intro class.

Grades will be based on attendance and other class participation. There will be lists of homework problems passed out sporadically and posted on the course webpage; you may do them or not, as you choose, but as you have already experienced, you will learn more if you think about the problems. If you wish to do so, you may turn in any problems you have done and I will read your work and add comments for your consideration.

Resources

There will not be an "official" textbook for this course and you will *not* be required to purchase any books as several books will be on reserve in the library and may be consulted there. No assignments will be made from any books.

The following books cover some of the material that is central, in my view, to this course, but they each have different perspectives the general subject and have chosen different topics to include in their books. They may be useful references for the course. The books marked with an asterisk(*) will be on reserve for this course in the IUPUI library.

J. B. Conway	*A Course of Functional Analysis**
J. B. Conway	*A Course in Operator Theory*
R. G. Douglas	Banach Algebra Techniques in Operator Theory
R. G. Douglas	Banach Algebra Techniques in the Theory of Toeplitz Operators
R. E. Megginson	*Introduction to Banach Space Theory**
A. Taylor and D. Lay	*Introduction to Functional Analysis*
P. R. Halmos	*Introduction to Hilbert Space* $\cdots$*
P. R. Halmos	*Hilbert Space Problem Book**
W. Rudin	*Functional Analysis**

The following books are classics, in my view, and might be useful references, but none of these books is on reserve in the library.

Dunford and Schwartz	*Linear Operators* (3 Volumes)
Riesz and Sz. Nagy	*Functional Analysis*
K. Yosida	*Functional Analysis*

Campus Course Policies

IUPUI has certain policies that apply to every course; this course will follow these policies also. You should be familiar with the policies; they may be found at

`http://registrar.iupui.edu/course_policies.html`