Math 55400: Linear Algebra (Class No: 22353)

Meets: MW 12:00 – 1:15p in IT 065
Final Exam: Wednesday, May 3, 10:30a – 12:30p
Prerequisite: Math 35100 or Math 51100 or equivalent undergrad linear algebra course

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Office Hours: M 1:35-3:00p, Tu 9:30-11:00a, W 3:10-4:00p, or by appointment
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URL: `http://www.math.iupui.edu/~ccowen/Math554.html`
Linear Algebra Student Seminar: Thursdays, 1:30-2:45p in IT 065

Linear algebra is second only to calculus in terms of importance for applications. In many applications, the problem is formulated mathematically, it is then converted to a linear algebra problem (possibly without the user knowing it), the linear algebra problem is solved using a computer, and, finally, the results are interpreted. For example, many numerical routines for solving differential equations change the problem into a linear algebra problem first. In addition, often the first step in solving non-linear differential equations is the solution of a 'linearization' of the actual problem. Although much could be done in a course called 'Linear Algebra' to learn more about the connections to these ideas, this course is not an applied linear algebra course and we will not discuss most of these ideas.

In addition, linear algebra and the types of arguments typical in linear algebra are important in many areas of 'pure' mathematics, including analysis, geometry, and algebra. The fact of linear algebra forming a foundation for much of the mathematics of the 20th and 21st centuries is the motivation for this course and the requirement that it be a part of the PhD program. The principal goal of this course is to help you develop an understanding of the facts and methods of proof in linear algebra. In addition, there will be an introduction to some aspects of algebras of matrices and algebras of polynomials.

The focus of the course will be theorems and proofs related to linear algebra, but there will inevitably be some computations that are necessary as well. You will need to be proficient in doing these computations by hand, at least in low dimensions, because the tests in this course and the qualifying exams are answered without computing devices. On the other hand, you are 21st century citizens and mathematicians, so it is not reasonable for you to be ignorant of computer algebra systems like *Mathematica*® or *Maple*® and numerical computational engines like *Matlab*®. Throughout the course, you should remain conscious of the reliance on computers for real world computation. You are welcome to do computations in your homework using machines, and if you do so, 'show your work' means telling what you asked the computer to do and then you report the answer. (No printouts may be handed in!!!)

This is a graduate mathematics course, although (because it is a "dual level" course) mathematically mature undergraduates with the appropriate background are welcome. This course is not at the research level, but it is expected that students taking this course are preparing for research in mathematics or a related discipline such as computer science, engineering, or physics. Indeed, this course is one of the 'Core 4' subjects and it should provide much of the background in preparation for the Qualifying Examination in Linear Algebra in the IUPUI mathematics PhD program. The grading policies (see below) have been chosen to reinforce these goals.

The necessary background for success in this course is at least a semester of linear algebra. While this class will cover all of the theory included in such a course, it will be difficult to keep up if you do not have any background from an undergraduate level course. All of the necessary background (and more) is included in the book *Linear Algebra for Engineering and Science* by Cowen, 1996, which is on reserve in the library for your use.

TEXT: The official text for the course is *Linear Algebra* (2^{nd} (or other) edition) by Hoffman and Kunze, about $30 (hardcover) from Amazon for some editions, $712 for another(!). You will *not* be required to purchase any books, as several books will be on reserve in the library and may be consulted there. No assignments will be made from any books.

Topics

- Fields, matrices, and systems of linear equations
- Vector spaces, subspaces, bases and dimension
- Linear transformations, matrices for linear transformations, linear functionals
- Polynomials, algebras, ideals
- Commutative rings, determinants
- Eigenvalues, annihilating polynomials, invariant subspaces, simultaneous triangularization and diagonalization. Primary Decomposition Theorem
- Rational and Jordan Canonical Form Theorems
- Inner products, inner product spaces, adjoints, unitary and normal operators
- Forms on inner product spaces, spectral theory

∗ *(if time permits)* Bilinear forms

Grading policy and philosophy

3 Keys to Learning Mathematics
1. Work lots of problems.
2. Memorize definitions and the statements of major theorems.
3. Work lots more problems.

Ordinarily, after mathematicians and scientists complete research on a topic, they write up their results and submit them for publication to an appropriate journal. To be accepted for publication, the work must be interesting, correct, and reasonably well written.

For the purposes of this class, all homework problems are, by hypothesis, interesting. Part of what you should be learning in this class, and others at this level, is how to write proofs that are both correct and well enough written to be understood by someone who does not know the result you are writing about. At this time, the international language of mathematics is English, so you will benefit by knowing how to communicate well in English. Therefore, your graded work will have comments about your writing and grammar in addition to comments about your mathematical ideas.

Weekly homework will be assigned (by being posted on the course webpage) and some of it will be identified (∗) when assigned to be collected and looked over. One problem (∗∗) from this list will be handed in separately, graded with more care, and be permitted to be resubmitted, one time, after correction. There will be one quiz, January 18, and the grade on the quiz and the graded homework will constitute about 30% of your final grade. There will be one midterm test, March 1, counting about 30% and a (cummulative) final exam on May 3 that will count about 40% of your grade.

Midterms and the Final Exam: The Midterm and the Final Exam will consist partly of questions concerning definitions, theorems, and examples from the course. You will *not* be asked to repeat proofs of the theorems from the course on the midterm or the exam. For example, "State the Rank-Nullity Theorem and find bases for the range and null space of the given 3×5 matrix A." would be a possible question on a test in this course, but "Prove the Rank-Nullity Theorem." would not be. Problems from the assigned homework in this course, problems from the text or other reference books, or problems from the qualifying exam archive on the department website will be considered reasonable sources for questions on the midterm or the final exam. One of the goals of the midterm and the final exam will be to help you to become accustomed to the kind of tests that constitute the qualifying exams.

General Academic Policies: IUPUI has certain policies that apply to every course; this course will follow these policies also. You should be familiar with the policies, especially those pertaining to academic integrity and adaptive services; they may be found at

`http://registrar.iupui.edu/course_policies.html`
`http://registrar.iupui.edu/withdrawal-policy.html`
and `http://registrar.iupui.edu/misconduct.html`

More information concerning adaptive educational services for learning or other disabilities at IUPUI can be found at `http://aes.iupui.edu/`

Some Important Dates

Date	
January 9	First Day of Classes
March 1	Midterm Test
March 10	Last day to withdraw with adviser's signature and automatic "W" (Withdrawal after March 10 requires Dean's approval, which is rarely given)
March 11 to March 19	Spring Break!! no classes!
May 1	Last Day of Classes
May 3	Final Exam, 10:30a–12:30p