April 24

$**$ **103.** Let $A = \begin{pmatrix} 1 & 3 & 3 & 3 & 0 & -1 \\ 2 & 0 & 7 & 0 & -8 & -3 \\ -1 & -2 & -3 & -2 & 1 & 1 \\ -2 & 1 & -6 & 1 & 7 & 2 \\ -1 & -1 & -3 & -1 & 2 & 1 \\ 1 & 0 & 3 & 0 & -3 & -1 \end{pmatrix}$

(a) Show that A is nilpotent and find the order of nilpotence for A.

(b) Find a similarity matrix S so that $J = S^{-1}AS$ is upper triangular with 0's on the diagonal and the super-diagonal of J consists of 0's and 1's and the entries of J are zeros except on the superdiagonal.

$*$ **104.** Let $B = \begin{pmatrix} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 1 & 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 0 & 1 \end{pmatrix}$

Considering B as a matrix with entries in the field $\mathbb{C}$, the minimal polynomial of B is
$p(x) = x^4 - 2x^3 + x^2$ and the characteristic polynomial is $q(x) = x^5 - 3x^4 + 3x^3 - x^2$.
Find a complex matrix A in Jordan canonical form that is similar to B.

$*$ **105.** Let C be a 5×5 matrix with characteristic polynomial is $p(x) = (x-2)^3(x+7)^2$ and minimal polynomial $q(x) = (x-2)^2(x+7)$.
Find a matrix in Jordan canonical form that is similar to C.

106. (a) Classify up to similarity all 3×3 matrices over $\mathbb{C}$ that satisfy $A^3 = I$. (Justify!!)

(b) Classify up to similarity all 4×4 matrices over $\mathbb{C}$ that satisfy $A^4 = I$. (Justify!!)

107. (a) Suppose N is a $k \times k$ matrix over $\mathbb{C}$ that satisfies $N^k = 0$, but $N^{k-1} \neq 0$.
Prove that N is similar to its transpose, N^t.

(b) Use Jordan Canonical Form and part (a) to show that all $n \times n$ complex matrices are similar to their transposes.

$*$ **108.** Prove that an orthogonal set of non-zero vectors is linearly independent.

$*$ **109.** Let $A = \begin{pmatrix} 1 & 1 & -1 & 2 \\ -1 & 0 & 2 & -3 \\ 1 & -1 & -3 & 4 \end{pmatrix}$

The vectors $v_1 = (2, -1, 1, 0)$ and $v_2 = (-3, 1, 0, 1)$ are a basis for the nullspace of A.

(a) Find a basis for the range of A.

(b) Find a basis for the range of A'.

(c) Find a basis for the orthogonal complement of the range of A'.

(d) Find a basis for the nullspace of A'.