March 20

$*$ **60.** Let $\mathcal{V}$ be the vector space of polynomials in $\mathbb{R}[x]$ with degree 3 or less. Let a and b be fixed real numbers and let f be the linear functional on $\mathcal{V}$ defined by $f(p) = \int_a^b p(x)\,dx$
Let D be the differentiation operator on $\mathcal{V}$. Find $D^t f$.

$*$ **61.** Let n be a positive integer and let $\mathcal{W}$ be the vector space of polynomials in $\mathbb{R}[x]$ with degree n or less. Let D be the differentiation operator on $\mathcal{W}$. Find a basis for the null space of D^t.

$*$ **62.** Prove that an upper triangular $n \times n$ matrix has determinant the product of the diagonal elements.

$*$ **63.** Let n be a positive integer and let $a_1, a_2, a_3, \cdots, a_n$ be scalars in the field F.
Prove that a Vandermonde matrix

$$\begin{pmatrix} 1 & a_1 & a_1^2 & \cdots & a_1^{n-1} \\ 1 & a_2 & a_2^2 & \cdots & a_2^{n-1} \\ 1 & a_3 & a_3^2 & \cdots & a_3^{n-1} \\ \vdots & \vdots & & \ddots & \vdots \\ 1 & a_n & a_n^2 & \cdots & a_n^{n-1} \end{pmatrix} \quad \text{has determinant} \quad \prod_{1 \le i < j \le n} (a_j - a_i)$$

$**$ **64.** (a) Write out the 24 permutations of the integers 1 to 4 and classify each permutation as odd or even.

(b) We know that $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$

Use the signs of the permutations given in part (a) to write the similar formula for the determinant of the 4×4 matrix A, below, in terms of sums of signed products of entries:

$$\text{If} \quad A = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ k & l & m & n \\ p & q & r & s \end{pmatrix} \quad \text{then} \quad \det(A) = ??$$