March 6

$*$ **56.** Find the g.c.d. of each of the following pairs of polynomials.

(a) $3x^4 + 8x^2 - 3$ and $x^3 + 2x^2 + 3x + 6$.

(b) $x^4 - 2x^3 - 2x^2 - 2x - 3$ and $x^3 + 6x^2 + 7x + 1$.

$*$ **57.** Let p be a monic polynomial over the field F and let h be the g.c.d. of the polynomials f and g in $F[x]$. Find the g.c.d. of the polynomials pf and pg.

$*$ **58.** Use the Lagrange interpolation formula to find a polynomial f with real coefficients and degree no more than 3 such that $f(-1) = -6$, $f(0) = 2$, $f(1) = -2$, and $f(2) = 6$.

Recall: The binomial coefficients are $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$ where $n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$ and $0!$ is defined to be $0! = 1$.

The Binomial Theorem is $(x+y)^n = \displaystyle\sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$

$**$ **59.** Let F be a field and let f be in F^∞, that is, f is a formal power series with coefficients in F. In analogy with evaluating polynomials at scalars from F, for f in F^∞ and a in F, define $f(a)$ in F^∞ by:

For $f = (f_0, f_1, f_2, f_3, \cdots)$ let $f(a) = (f_0, f_1 a, f_2 a^2, f_3 a^3, f_4 a^4, \cdots)$

For F a subfield of $\mathbb{C}$, we define the function *exp* for a in F, to be $exp(a)$ is the formal power series

$exp(1) = (1, 1, (2!)^{-1}, (3!)^{-1}, \cdots)$ and $exp(a) = (1, a, a^2/2!, a^3/3!, a^4/4!, \cdots)$

Using the definition of products in F^∞ and the binomial theorem, prove that, for a and b in F,

$exp(a)exp(b) = exp(a+b)$