February 22 NO CLASS on February 22: Put solutions into Cowen's mailbox in LD 270

$*$ **46.**

(a) Let f be the polynomial $f(x) = x^3 - 4x^2 + 3x - 5$. Let B be a 3×3 invertible matrix that satisfies $f(B) = 0$. Find a polynomial g so that $B^{-1} = g(B)$.
(Hint: rewrite the equation $f(B) = 0$ in such a way as to get I alone on the right side of the equation.)

(b) In Exercise 29, it was shown that every 3×3 matrix, A, satisfies a polynomial equation $p(A) = 0$ for some non-zero polynomial. The same can be done for $n \times n$ matrices: If A is an $n \times n$ matrix, there is a non-zero polynomial p for which $p(A) = 0$. Assuming that result has been proved, show that for every invertible $n \times n$ matrix A, there is a polynomial q so that $A^{-1} = q(A)$.

$*$ **47.** Use the Lagrange interpolation formula to find a polynomial f with real coefficients and degree no more than 3 such that $f(-1) = -6$, $f(0) = 2$, $f(1) = -2$, and $f(2) = 6$.

$*$ **48.** Let n be a positive integer and F a field. Suppose A is an $n \times n$ matrix over F and P is an invertible $n \times n$ matrix over F. Prove: if f is any polynomial over F, then

$$f(P^{-1}AP) = P^{-1}f(A)P$$

$*$ **49.** Let $\mathbb{Q}$ the field of rational numbers. Determine which of the following are ideals in $\mathbb{Q}[x]$. If the set is an ideal, find a monic generator. If it is not an ideal, explain why it is not.

(a) All polynomials with even degree.

(b) All polynomials f with degree$(f) \geq 5$.

(c) All polynomials f for which $f(2) = f(4) = 0$.

(d) All polynomials f for which $f(2) - f(4) = 0$.

(e) The range of the linear transformation $T(f) = (5x^2 + 2)f$.

$*$ **50.** Find the g.c.d. of each of the following pairs of polynomials.

(a) $3x^4 + 8x^2 - 3$ and $x^3 + 2x^2 + 3x + 6$.

(b) $x^4 - 2x^3 - 2x^2 - 2x - 3$ and $x^3 + 6x^2 + 7x + 1$.

$*$ **51.** Let F be a subfield of the complex numbers.

(a) Let A be an $n \times n$ matrix over F.
Prove: the set of polynomials in $F[x]$ for which $f(A) = 0$ is an ideal.

(b) Find the monic generator of the ideal of polynomials in $F[x]$ for which $f(A) = 0$ when

$$A = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$$

$*$ **52.** Let p be a monic polynomial over the field F and let h be the g.c.d. of the polynomials f and g in $F[x]$. Find the g.c.d. of the polynomials pf and pg.