February 1

$*$ **18.** Consider the homogeneous system of equations

$$(*)\quad \begin{cases} 2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0 \\ x_1 + \frac{2}{3}x_3 - x_5 = 0 \\ 9x_1 - 3x_2 + 6x_3 - 3x_4 - 3x_5 = 0 \end{cases}$$

Let W be the subspace of $\mathbb{R}^5$ consisting of those vectors that are solutions of system $(*)$. Find a basis for W.

$*$ **19.** Let W_1 and W_2 be subspaces of the vector space $\mathcal{V}$ such that their set theoretic union, $W_1 \cup W_2$, is a subspace of $\mathcal{V}$. Prove that either $W_1 \subset W_2$ or $W_2 \subset W_1$.

$*$ **20.** Let M_1 and M_2 be subspaces of the vector space $\mathcal{V}$ such that $M_1 + M_2 = \mathcal{V}$ and $M_1 \cap M_2 = (0)$. Prove that every vector v in $\mathcal{V}$ can be written as $u_1 + u_2 = v$ where u_1 is a vector in M_1 and u_2 is a vector in M_2 and that u_1 and u_2 are the only vectors in M_1 and M_2, respectively, for which this is true.

$*$ **21.** Let $\mathcal{V}$ be the vector space of 2×2 matrices over the field F. Find a basis for $\mathcal{V}$ consisting of matrices A_1, A_2, A_3, A_4 such that $A_j^2 = A_j$ for each of $j = 1, 2, 3,$ and 4.

$*$ **22.** Let A be an $m \times n$ matrix with $m < n$. Show that the system of equations $AX = 0$ has a non-trivial solution.

$*$ **23.** Find a homogeneous system of equations such that subspace of solutions of the system is spanned by $u = (-1, 0, 1, 2)$, $v = (3, 4, -2, 5)$, and $w = (1, 4, 0, 9)$.

$*$ **24.** Show that the vectors $z_1 = (1, 0, -i)$, $z_2 = (1+i, 1-i, 1)$, and $z_3 = (i, i, i)$ comprise a basis for $\mathbb{C}^3$ and write the vector (a, b, c) as a linear combination of z_1, z_2, and z_3.

$*$ **25.** Let V be the subspace of $\mathbb{R}^5$ spanned by the rows of the matrix

$$A = \begin{pmatrix} 3 & 20 & 0 & 9 & 0 \\ 1 & 7 & -1 & -2 & -1 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{pmatrix}$$

(a) Find a basis for V.

(b) Find a matrix B with 5 rows so that X is in V if and only if $XB = 0$.