April 19

∗ **95.** Let T be the linear transformation on $\mathbb{C}^3$ whose matrix with respect to the usual basis is

$$\begin{pmatrix} 1 & i & 0 \\ -1 & 2 & -i \\ 0 & 1 & 1 \end{pmatrix}$$

(a) Find the T-annihilator of $(1, 0, 0)$.

(b) Find the T-annihilator of $(1, 0, i)$.

∗ **96.** Let T be a linear transformation on the finite dimensional vector space $\mathcal{V}$.

(a) Prove that if T^2 has a cyclic vector, then T has a cyclic vector.

(b) Is the converse true? Either give a proof or a counterexample to show that your answer is correct.

∗ **97.** Let N be a nilpotent linear transformation on the n-dimensional vector space $\mathcal{V}$.

(a) Prove: N has a cyclic vector if and only if $N^{n-1} \neq 0$.

(b) If v is a vector in $\mathcal{V}$ for which $N^{n-1}v \neq 0$, what is the matrix for N with respect to the basis v, Nv, $\cdots$, $N^{n-1}v$.

∗ **98.** A linear transformation acting on $\mathbb{R}^3$ has matrix with respect to the usual basis:

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ -3 & -3 & -5 \end{pmatrix}$$

Find a 3×3 matrix P such that $P^{-1}AP$ is in rational form.

∗ **99.** Prove that if A and B are 3×3 matrices over the field F, then A and B are similar if and only if they have the same minimal polynomials and the same characteristic polynomials. Give an example that shows this is not a theorem for 4×4 matrices.

∗ **100.** Let C be a linear operator on a finite dimensional vector space $\mathcal{V}$.

(a) Prove: If C does not have a cyclic vector, there is an operator G that commutes with C, but G is not a polynomial in C.

(b) Prove: If C has a cyclic vector, every operator that commutes with C is a polynomial in C.

In other words, C has a cyclic vector if and only if every operator that commutes with C is a polynomial in C.

∗ **101.**

Let $$B = \begin{pmatrix} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 1 & 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 0 & 1 \end{pmatrix}$$

Considering B as a matrix with entries in the field $\mathbb{C}$, the minimal polynomial of B is $p(x) = x^4 - 2x^3 + x^2$ and the characteristic polynomial is $q(x) = x^5 - 3x^4 + 3x^3 - x^2$. Find a complex matrix A in rational canonical form that is similar to B.