Professor Carl Cowen **Math 55400** **Spring 17**

NOTES and Comments on Inner Products and Operators on Inner Product Spaces

Throughout this document, $\mathcal{V}$ will be a finite dimensional vector space over the field $\mathbb{C}$ or $\mathbb{R}$, u, v, w, etc., will be vectors in $\mathcal{V}$, S, T, etc., will be linear transformations/operators acting on $\mathcal{V}$ and mapping into $\mathcal{V}$, and A, B, C, etc., will be real or complex matrices, but might also be considered the transformation on $\mathbb{C}^n$ that has the given matrix as its associated matrix with respect to the usual basis for $\mathbb{C}^n$. The symbol I will represent the identity transformation or the identity matrix appropriate to the context. References will be to *Linear Algebra for Engineering and Science*, Cowen, 1995. After the notes and comments section, there is a list of important things from that book, which I hope to cover in class, to know for the course Final Exam, as well as the Qualifying Exams I might write. At the end of this document, there is a list of *un-starred!* problems from that book that might help review material from your undergraduate course.

- **Section 4.2:** Most of the material in this section are things I would expect to have been in your undergrad course.
- **Section 4.3:** I would expect Gram-Schmidt orthogonalization to have been in your undergrad course. Probably QR–factorization was not in your undergrad course, and we will not discuss it either. The take-away message, however, is important: In any inner product space, for any well-ordered spanning set for a subspace W, applying the Gram-Schmidt algorithm leads to a well-ordered orthonormal basis for W.

Core Material

- **Section 4.4:** Definition of *orthogonal complement*, Thm. 4.18 on properties of orthogonal complements, Thm. 4.19 relationships between ranges and null spaces of matrices in an inner product space.
- **Section 6.1:** Thm. 6.1, for any subspace W, decomposition of $\mathcal{V}$ into a direct sum of W and $W^{\perp}$; closest points. Definition of orthogonal projections. Thm. 6.5 orthogonal projection onto a subspace. Thm. 6.6 characterization of orthogonal projections.
- **Section 9.1:** Definitions of unitary, Hermitian (self-adjoint, symmetric). Thm. 9.4 Schur's Triangularization Theorem
- **Section 9.2:** Thm. 9.5, 9.6 eigenvalues and eigenvectors of Hermitian matrices Thm. 9.7 unitary diagonalization of Hermitian matrices and corollaries.
- **Section 9.3:** Definition of normal matrices. Thm. 9.14 eigenvalues and eigenvectors of normal matrices. Thm. 9.17 spectral theorem for normal matrices.

Suggested Review (no-star) Problems

Page 193, Section 4.2: 22, 23, 24, 29

Page 201, Section 4.3: 5

Page 213, Section 4.4: 17, 18, 19

Page 252, Section 6.1: 3, 4, 5, 7

Page 343, Section 9.1: 1, 2, 7, 8

Page 352, Section 9.2: 1, 2, 3ab, 4, 6

Page 358, Section 9.3: 2, 3, 6