Professor Carl Cowen **Math 53000** **Spring 2014**

OUTLINE to February 13

- The complex numbers: field properties, conjugate, absolute value (modulus), real and imaginary parts
- Geometry of complex numbers: metric for $\mathbb{C}$, argument, DeMoivre's Formula
- Riemann sphere/extended complex plane: stereographic projection of sphere onto $\mathbb{C}$
  - lines in the plane as circles through ∞ on the sphere
- Definition of "function $f : G \mapsto \mathbb{C}$ is differentiable" for G an open set in $\mathbb{C}$.
- Calculus:
  - f differentiable implies f continuous
  - sum, product, quotient rules for derivatives
  - chain rule for derivatives
  - Cauchy-Riemann equations
- Harmonic functions
- Examples of holomorphic/analytic functions:
  - polynomials in the complex variable z
  - rational functions in the complex variable z
  - linear fractional maps
- Power series
  - basic properties: open regions of absolute convergence/divergence, radius of convergence
  - differentiability of a power series in the open disk of absolute convergence
  - series for derivatives (of all orders) of a power series and their radii of convergence
  - exponential function $\exp(z)$
  - trigonometric functions $\cos(z)$, $\sin(z)$, etc.
  - properties of exponential and trigonometric functions
- Branches of inverses of holomorphic functions
  - log as inverse of exponential function
  - $\sqrt{z}$, $\sqrt[3]{z}$, etc. as inverses of polynomials
  - definition of a^c for complex numbers a and c with $a \neq 0$

- Holomorphic functions as mappings
  - Linear fractional maps as univalent mappings of the Riemann sphere onto itself
    - take circles(lines) to circles(lines)
    - transitivity property: any three points in plane can be taken to any other three points by a linear fractional map
  - Polynomials and rational functions of degree n as n - to - 1 coverings of $\widehat{\mathbb{C}}$ onto $\widehat{\mathbb{C}}$
  - Exponential function as an ∞ - to - 1 covering of $\mathbb{C}$ onto $\mathbb{C} \setminus \{0\}$
- Principle of conformal mapping:
  - If f is holomorphic in a domain G, the point z_0 is in G, the derivative $f'(z_0) \neq 0$, and γ_1 and γ_2 are curves in G that both pass through z_0, then the angle between γ_1 and γ_2 at z_0 is the same as the angle between the curves $f(\gamma_1)$ and $f(\gamma_2)$ at $f(z_0)$. Specifically, the argument of the line tangent to the curve $f(\gamma_j)$ at $f(z_0)$ is the sum of the argument of the line tangent to the curve γ_j at z_0 and the argument of $f'(z_0)$ for each of $j = 1$ and $j = 2$.