Constrained Maxima and Minima

1. Let $K = \{\binom{x}{y} : x^2 + 2y \leq 5 \text{ and } y \geq -1\}$ and let $f\binom{x}{y} = 2x^2 - 2xy + y^2 - 4x + 2y$.

Find the maximum and minimum values of f on K and the points at which these values occur.

2. Let $L = \{\begin{pmatrix} x \\ y \\ z \end{pmatrix} : x^2 + 4y^2 + z^2 + 6z \leq 31\}$ and let $F\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2x + 4y^2 + 4z$.

Find the maximum and minimum values of F on L and the points at which these values occur.

Section 4.1 of text

3. Exercise 4.1.1 (page 406 (404)).

4. Let $f(x) = \begin{cases} 0 & x \leq 1 \\ x - 1 & 1 < x < 3 \\ 2 & 3 \leq x \end{cases}$ and let $g(x) = f(x)f(10 - x)$.

(a) Graph the functions f and g.

(b) Is f continuous on $\mathbb{R}$? Is g continuous on $\mathbb{R}$?

(c) Find the support of f. Is the support of f a bounded set?

(d) Find the support of g. Is the support of g a bounded set?

5. Let $A = \{\binom{x}{y} : x^2 - 4x + y^2 \leq 0\}$ and let $\mathbf{1}_A$ be the characteristic function of A in $\mathbb{R}^2$.

(a) Consider the "0^{th} dyadic paving" of $\mathbb{R}^2$ defined in the text. Which of the "dyadic cubes" in this paving has non-empty intersection with A?

(b) How many of the cubes in the 1^{st} dyadic paving of $\mathbb{R}^2$ intersect A.

(c) Find the 0^{th} upper sum, $U_0(\mathbf{1}_A)$ and the 0^{th} lower sum $L_0(\mathbf{1}_A)$.

(d) We will see that A is a "pavable set" in the language of the text.

What is $\displaystyle\int_{\mathbb{R}^2} \mathbf{1}_A \, |d^2x| \, ?$