Math 51000 (Cowen) **Handout 1** **11 January 2010**

For discussion Wednesday, 13 January:

Please look over the material in calculus and linear algebra books to refresh your memory, if necessary, in answering these questions. Please do the following problems; they will not be collected, but please be prepared to ask any questions you might have on the material related to these questions and to ask questions about the ones you find difficult.

1. Let $r(x) = \begin{cases} \frac{2x^2-5x-3}{x^2-4x+3} & x \neq 3 \\ c & x = 3 \end{cases}$

   where c is a number.

   (a) Find $\lim_{x\to 3} r(x)$.

   (b) Discuss the continuity of r at $x = 3$.

2. In the following, a and b are fixed numbers. Differentiate with respect to x:

   $(a)\quad f(x) = \frac{4x^2+2x-7}{\sin 3x}$ $\qquad$ $(b)\quad g(x) = \frac{2ax^2+ax-7}{\sin bx}$

   $(c)\quad h(x) = e^{3x^2-5x+6}$ $\qquad$ $(d)\quad j(x) = e^{ax^2+bx+6}$

3. The curve determined by the equation $y^2 - 4x^2 = 4y + 8x + 5$ passes through the point $(0, -1)$ and, near that point, determines y as a function of x.

   (a) Use implicit differentiation to find the equation of the line tangent to the curve at the point $(0, -1)$.

   (b) Find, explicitly, the function $y(x)$ determined by the curve and satisfying $y(0) = -1$. Then, find the equation of the line tangent to the graph of this function at the point $(0, -1)$. Is your answer the same as in part (a)?

4. Consider the integral $\displaystyle\int_{-2}^{7} x^2\sqrt{x+2} + 3x\, dx$.

   (a) Use the substitution $x = t^2 - 2$ to transform the given integral into a new, equivalent, integral in t.

   (b) Use your answer to part (a) to find the value of the given integral.

5. Solve the following system. If there are no solutions, say so; if the solution is unique, say so; if there are infinitely many solutions, find the general solution and give two solutions explicitly.

$$\begin{cases} w \quad\;\; + 2y + \;\; z = -2 \\ -w + x - \;\; y + 3z = -1 \\ 2w - x + 2y - 3z = 1 \\ w + x - \;\; y + \;\; z = 3 \end{cases}$$

6. Find the determinant of $A = \begin{pmatrix} 0 & 1 & 1 & -2 \\ 1 & 1 & 4 & -2 \\ 1 & -1 & 0 & 6 \\ 2 & -1 & 1 & 3 \end{pmatrix}$

7. (a) Find the inverse of $B = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -3 & 0 \\ 1 & 1 & 3 \end{pmatrix}$

   (b) Show in a way different from doing the computation over again that your answer is correct.

8. Let $C = \begin{pmatrix} 1 & 0 & 2 & 1 & 1 \\ -1 & 1 & -1 & 3 & 1 \\ 2 & -1 & 2 & -3 & 0 \\ 1 & 1 & -1 & 1 & 3 \end{pmatrix}$

   (a) Find a basis for the kernel (nullspace) of C.

   (b) Find a basis for the image (range) of C. (Show explicitly that the vectors you chose are linearly independent.)

9. Let $r = (1, 3)$, $s = (-2, 5)$, $u = (1, -2, 1)$, $v = (1, 1, -4)$, and $w = (0, -2, 3)$. Find

(a) $r \cdot s$	(b) $\|s\|$	(c) $u \cdot v$
(d) $u \times v$	(e) $v \cdot (u \times v)$	(f) $u \cdot (v \times w)$
(g) $(u \times v) \times w$	(h) $u \times (v \times w)$	(j) $w \times (u \times v)$

10. Suppose u and v are vectors in $\mathbf{R}^n$. The geometric figure whose sides are (two copies of) u and v is a parallelogram in $\mathbf{R}^n$. A sketch will convince you that the vectors $u+v$ and $u-v$ are the diagonals of the parallelogram. The geometry induced by the dot product corresponds to ordinary Euclidean geometry; here are two theorems from high school geometry that you can prove using dot products:

    (a) Prove the 'Parallelogram Law' (the sum of the squares of the lengths of the sides of a parallelogram is the sum of the squares of the lengths of the diagonals):

    $$2\|u\|^2 + 2\|v\|^2 = \|u + v\|^2 + \|u - v\|^2$$

    (b) Recall that a rhombus is a parallelogram whose sides have equal length. Prove that the diagonals of a rhombus are perpendicular to each other, that is, prove that if $\|u\| = \|v\|$, then $(u + v) \cdot (u - v) = 0$.