Math 44500: Foundations of Analysis II (Class No: 25300)

Meets: TuTh 4:30–5:45p in SL 055
Final Exam: Tuesday, May 3, 3:30–5:30p

Instructor: Carl Cowen **Office:** LD 224P **Phone:** 278-8846
Office Hours: TuWTh 3:00–4:00p, or by appointment
E-mail: ccowen@math.iupui.edu
URL: http://www.math.iupui.edu/~ccowen/Math445.html

General Information and Goals

Math 44400 and Math 44500 together form a foundation for analysis, both as a rigorous treatment of many topics in calculus and as an introduction to the mathematical area of analysis generally and the analysis of real-valued functions of a real variable specifically. Many of the topics in these courses came about in response to deepening understanding of the subject over the course of the decades following the development of calculus to the formalization of many of the basic ideas at the end of the 19th century. This formalization formed an important backdrop and motivation for the changing view of rigor and the foundations of mathematics generally that were important in 20th century mathematics. Real analysis, including much of the material of these courses, is at the heart of much of modern mathematics and forms essential background for the understanding of these subjects and their applications in other parts of analysis, differential equations, probability and statistics, dynamical systems, mathematical physics, computational and applied mathematics, as well as subjects such as engineering, economics, physics, and biology that depend on mathematics. Math 44500 is a sequel to Math 44400; it is more abstract than Math 44400 and it both fills in some gaps and extends the ideas of Math 44400. Many graduate programs in mathematics or statistics, and sometimes other areas, expect students to know the material in both Math 44400 and 44500.

In the past, both Math 44400 and Math 44500 were required of "Pure Math" majors, whereas only Math 44400 was required of "Applied Math" majors. Beginning in 2010, the requirements for "Pure Math" have changed and now Math 44400 is part of a list of three courses, two of which are required, but also *2 two-course sequences* are required for the "Pure Math" major. Math 44400 and 44500 form one *two-course sequence*, but there are ten or so others (such as Math 35100 and 35300 or Math 45300 and 45600 or Math 32100 and 57100) that can be used. A document describing the new requirements for "Pure Math", including a complete list of the "two-course sequences" is at www.math.iupui.edu/program/checksheets/CheckSheetPM.pdf as well as a link from the course webpage (above).

The official text will be

Text: *Introduction to Real Analysis*, by Robert G. Bartle and Donald R. Sherbert, Wiley, 3^{rd} Edition (2000) (ISBN 0-471-32148-6)

The topics covered in this course will be covered, although not necessarily in the same way or in the same depth, by almost any book whose title is "Real Analysis", for example, the books listed below. Those on reserve in the library are marked with "(∗)".

(∗) *Introduction to Real Analysis*, by Robert G. Bartle and Donald R. Sherbert, Wiley, 3^{rd} Edition (2000)

(∗) *The Elements of Real Analysis*, by Robert G. Bartle, Wiley, 2^{nd} Edition (1976)

- *Real Analysis and Foundations*, by Steven G. Krantz, 2^{nd} Edition (2004)

(∗) *Real Analysis*, by H. L. Royden, 2^{nd} Edition (1988)

- *Closer and Closer: Introducing Real Analysis*, by Carol Schumacher, (2007)
- *Introduction to Real Analysis*, by William Trench, (2002) (Available FREE(!) on web)
- *Principles of Mathematical Analysis*, by Walter Rudin, 3^{rd} Edition (1976)
- *Understanding Real Analysis*, by Paul Zorn, (2010)

Reading and writing are important skills for life, including mathematics, and they will be important in this class as well. In your homework, your writing will be graded for spelling, grammar, and clarity of exposition as well as for mathematical correctness. In tests, writing is also important, but will not be as much of a focus as for the homework.

Reading will be an active part of this course as well. Reading assignments will be given and *your reactions to the reading assignments will be due by email by 2:00pm of the day specified in the assignment.* The tests and the final exam will also include readings that are related to the material of the course and questions concerning the reading will be part of the test.

Conversations about this mathematics will help facilitate your learning in the course, so occasional attendance at office hours is encouraged.

Homework, Test, Exam, and Grading Policies

In addition to the comprehensive Final Exam on May 3, there will be 2 tests during the semester.

Grades for the course will be based on the responses to the reading assignments (approximately 10%), written homework (approximately 20%), two midterm tests (approximately 20% each), and the comprehensive final examination (approximately 30%). Late homework assignments may be handed in for feedback if you wish, but they will be recorded in the gradebook as 0's and similarly, late reactions to the reading assignments will be read and may be commented on, but will be recorded as 0's. However, the lowest two homeworks and the lowest two reactions to the readings will be dropped before computing the final grades. Each homework assignment will be worth the same number of points and the reactions to the readings will be rated as *not returned or insubstantial response* (0 points), *fair* (1 point), or *good* (2 points).

In addition, there will a list of special problems, the *'A' List*, of longer, more interesting, and perhaps harder problems than those in the usual homework. Problems from this list

may be handed in at any time before 5:00pm on May 6. The problems will be read and either accepted as correct or returned for rewriting and resubmission. Only one of these problems will be counted for credit but this problem will be worth the same number of points as two regular homework assignments. In order to receive an 'A' or 'A+' for the course, you must have one of the *'A' List* problems accepted as correct. Problems will be added to this list as the semester progresses.

General Academic Policies

The work you submit for homework, tests, and the final exam must be your own. During tests and the final exam, no electronic devices, including calculators and cell phones, may be powered up or even visible and no books or notes are permitted.

For homework, you will probably find it beneficial to consult with other students about the material and this kind of conversation and collaboration is encouraged. At the end of the consultation, however, each participant is expected to prepare their own summary of the discussion and their own solution to the problem or project. The policies for this class will be those derived from IUPUI's policies on academic conduct and adaptive services. More information about student conduct can be found at

`http://registrar.iupui.edu/misconduct.html`

All students involved in a particular event of such dishonesty will receive a zero on the item involved; a second infraction by an individual would usually mean receiving an 'F' for the course.

More information concerning adaptive services for learning or other disabilities at IUPUI can be found at

`http://life.iupui.edu/aes/`

Some Important Dates

Date	
March 4	Last day to withdraw with adviser's signature and automatic "W"
March 14–18	Spring Break!! no classes
April 1	Last day to withdraw with permission of adviser and instructor
May 3	Final Exam, 3:30 – 5:30p

Approximate Course Outline

Section numbers refer to *Introduction to Real Analysis* by Bartle & Sherbert, 3^{rd} Edition **I** or *Elements of Real Analysis* by Bartle, 2^{nd} Edition **E**. (Primary reference listed first)

Section	*Topic*	*Lectures*
E 9, 11, 12; **I** 11.1,2	Revisit basic topological notions	2
I 4.3	Some extensions of the limit concept	3
E 21, 22; **I** 11.3	Revisit continuity of functions	2
E 23	Fixed points of continuous functions	1
I 5.6, 6.1	Monotone and inverse functions	3
	Midterm Test I (late February)	
I 6.3; **E** 28	L'Hôpital's Rules	1
I 6.4; **E** 28	Taylor's Theorem	2
I 8.1,2; **E** 24	Sequences of functions	2
I 7.1-3; **E** 28, 29, 30	Revisit Riemann integration	2
I 8.3	Exponential and logarithmic functions	2
	Midterm Test II (late March)	
I 9.1-3; **E** 34, 35, 36	Infinite series	2
E 37: **I** 9.4	Series of functions, power series	2
(hand-outs?)	Sets of measure zero	2
(hand-outs?)	Introduction to Lebesgue integration	1
	Review	1
	Final Exam Tuesday, May 3, 3:30 – 5:30p	