Math 44500 (Cowen) **Handout 8 (corrected)** **31 March 2011**

Due Thursday, 7 April:

From page 262 of **BS**: 6, 7, 14

From page 265 of **BS**: 1, 5 (see A below)

From page 272 of **BS**: 1, 2, 6

A. Recall, if $\sum_{n=1}^{\infty} x_n$ is a series, we say *the series* $\sum_{k=1}^{\infty} y_k$ *is obtained from the series* $\sum_{n=1}^{\infty} x_n$ *by grouping* if there are positive integers $j_1,\ j_2,\ j_3,\ \cdots$ with $1 \leq j_1 < j_2 < j_3 < \cdots$ such that $y_1 = \sum_{n=1}^{j_1} x_n,\ y_2 = \sum_{n=j_1+1}^{j_2} x_n,\ y_3 = \sum_{n=j_2+1}^{j_3} x_n$, etc. Recall also that we proved, "If $\sum_{n=1}^{\infty} x_n$ converges and the series $\sum_{k=1}^{\infty} y_k$ is obtained from the series $\sum_{n=1}^{\infty} x_n$ by grouping, then $\sum_{k=1}^{\infty} y_k$ also converges." but that the converse is false in general. Prove the following partial converse:

> Suppose (x_n) is a sequence of real numbers such that $\lim_{n\to\infty} x_n = 0$.
>
> If the series $\sum_{k=1}^{\infty} y_k$ is obtained from the series $\sum_{n=1}^{\infty} x_n$ by grouping, the series $\sum_{k=1}^{\infty} y_k$ converges, and there is an integer J so that $j_{k+1} - j_k \leq J$ for $k = 1, 2, 3, \cdots$ (so that the size of the 'groups' is bounded), then the series $\sum_{n=1}^{\infty} x_n$ converges.