Math 44500 (Cowen) **Homework 9** **Due April 2, 2015**

1. Consider *ternary* expansions of integers and rational numbers: The digits in a ternary expansion are 0, 1, or 2 and the number $a = (t_4t_3t_2t_1t_0.d_1d_2d_3)_3$ in base 3 is

$$a = t_4 3^4 + t_3 3^3 + t_2 3^2 + t_1 3^1 + t_0 3^0 + d_1 3^{-1} + d_2 3^{-2} + d_3 3^{-3}$$

(a) What number (as an integer and fraction in standard notation) is 2012.1_3?

(b) Use the fact that 3 is represented as 10_3, 4 is represented as 11_3 and long division to represent 3/4 in base 3.

(c) What number does $(.120120120120120\cdots)_3$ represent?

(d) What number does $(.2101222222222222222\cdots)_3$ represent?

(e) Explain why all rational numbers in $(0, 1)$, that is, numbers of the form p/q where p and q are positive integers with $p < q$, have ternary expansions that are repeating or terminating.

(f) Explain why no irrational numbers x with $0 < x < 1$ have repeating or terminating ternary expansions.