Math 44500 (Cowen) **Homework 4** **Due February 12, 2014**

A. (1) Prove that if S is a subset of $\mathbb{R}$ such that S is a set of measure zero (a null set) and T is a subset of S, then T is a set of measure zero also.

(2) Let S be a non-empty subset of $\mathbb{R}$ such that S is a set of measure zero. Prove that every connected subset of S is $\{p\}$ where p is a point of S.

B. Let a and b be real numbers with $a < b$ and suppose f is a continuous real-valued function on $[a, b]$. Define F on $[a, b]$ by $F(x) = \int_a^x f(t)\,dt$.

(1) For c with $a < c < b$, let $G(x) = \int_c^x f(t)\,dt$. Write G in terms of F.

(2) Find $G'(x)$ for $c < x < b$.

(3) For c with $a < c < b$, let $H(x) = \int_x^c f(t)\,dt$. Write H in terms of F.

(4) Find $H'(x)$ for $a < x < c$.

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Sections 11.3 and 11.4 are related to things covered in class relevant to these problems.

C. Let S and T be sets and let f be a function on S with values in T, that is, $f : S \mapsto T$, or for each s in S, $f(s)$ is a point of T.

Find an example of sets S and T and a function $f : S \mapsto T$ and subsets P and Q of S, such that $f(P) \cap f(Q) \neq f(P \cap Q)$

D. Let S and T be sets and let f be a function on S with values in T, that is, $f : S \mapsto T$:

Prove, for subsets U and V of T, that $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V)$

E. Let X be a metric space with metric d and suppose f is a function mapping X into itself, that is, for each x in X, $f(x) \in X$. Recall that we defined *the function f is continuous on X* if, for each open set U in X, the set $f^{-1}(U)$ is also open in X.

Prove: The function f is continuous on X if and only if for each point a in X and each sequence (x_n) such that $\lim_{n\to\infty} x_n = a$, we have $\lim_{n\to\infty} f(x_n) = f(a)$.