Math 44500 (Cowen) **Homework 3** **Due February 5, 2014**

A. As usual, we will regard $\mathbb{R}^2$ as a metric space with the distance function

$$d(p,q) = \|p - q\| = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2}$$

for $p = (p_1, p_2)$ and $q = (q_1, q_2)$. This means that a subset U of $\mathbb{R}^2$ is open if for every p in U, there is $\epsilon > 0$ so that the open ball $\{q \in \mathbb{R}^2 : d(p,q) < \epsilon\}$ is a subset of U.

Prove that a sequence $p_n = (p_{1,n}, p_{2,n})$ satisfies $\lim_{n\to\infty} p_n = q = (q_1, q_2)$ if and only if $\lim_{n\to\infty} p_{1,n} = q_1$ and $\lim_{n\to\infty} p_{2,n} = q_2$.

You may do **B.** and **C.** in either order, and you may use the truth of the first one you do in the proof of the second.

B. Let f be a continuous function on (a,b) for a and b real numbers with $a < b$.
Let $S = \{(x,y) \in \mathbb{R}^2 : a < x < b \text{ and } y > f(x)\}$. Prove that S is an open subset of $\mathbb{R}^2$.

As a corollary, deduce that the set $B = \{(x,y) \in \mathbb{R}^2 : a < x < b \text{ and } y < f(x)\}$ is also an open subset of $\mathbb{R}^2$.

C. Let g be a continuous function on $[a,b]$ for a and b real numbers with $a < b$.
Let $G = \{(x,y) \in \mathbb{R}^2 : a \le x \le b \text{ and } y = f(x)\}$. Prove that G is a closed subset of $\mathbb{R}^2$.

As a corollary, deduce that the set $H = \{(x,y) \in \mathbb{R}^2 : a \le x \le b \text{ and } y \le f(x)\}$ is also closed.

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