Math 44500 (Cowen) **Homework 2** **22 January 2014**

Due Thursday, 29 January: put in my mailbox in LD 270.

Please look over the material in section 11.1 and 11.2 of Bartle and Sherbert's book, most of which is familiar to you.

From page 332: 9, 10, 11, 12
From page 336: 1

A. Suppose A is a subset of $\mathbb{R}^n$. Show that a is a boundary point of A if and only if there is a sequence (a_n) consisting of points in A such that $\lim_{n\to\infty} a_n = a$ AND a sequence (b_n) consisting of points in the complement of A such that $\lim_{n\to\infty} b_n = a$.
(Note: constant sequences are OK.)

B. Suppose $K \subset \mathbb{N}$, regarded as a subset of $\mathbb{R}$. Show that K is compact if and only if K is finite.

C. Find a sequence $(x_n)_{n=1}^{\infty}$ of distinct real numbers, that is, if $n \neq m$, then $x_n \neq x_m$, so the set $\{x_n : n = 1, 2, 3, \cdots\}$ is a compact set.

Reminder: Definition of connected:
A set S is connected if there are NOT open sets U and V so that $U \cap V = \emptyset$, $U \cap S \neq \emptyset$, $V \cap S \neq \emptyset$, and $S \subset U \cup V$

D. Let $S = \{(x, y) \in \mathbb{R}^2 : x \leq 0 \quad \text{OR both} \quad y > 0 \text{ and } xy \geq 1\}$

(a) Prove that S is a closed set in $\mathbb{R}^2$.

(b) Show that S is not connected, that is, find open sets U and V in $\mathbb{R}^2$ so that $S \subset U \cup V$ and $U \cap S \neq \emptyset$, $V \cap S \neq \emptyset$, and $U \cap V = \emptyset$.