Math 44400: Foundations of Analysis I (Class No: 5311)

Meets: MW 6:00–7:15p in LD002
Final Exam: Wednesday, December 15, 6:00–8:00p

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Office Hours: M 3:00-4:00, Tu 4:00-5:00, W 1:45-2:45, or by appointment
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General Information and Goals

Math 44400 and Math 44500 together form a foundation for analysis, both as a rigorous treatment of many topics in calculus and as an introduction to the mathematical area of analysis generally and the analysis of real-valued functions of a real variable specifically. Many of the topics in these courses came about in response to deepening understanding of the subject over the course of the decades following the development of calculus to the formalization of many of the basic ideas at the end of the 19th century. This formalization formed an important backdrop and motivation for the changing view of rigor and the foundations of mathematics generally that were important in 20th century mathematics. Real analysis, including much of the material of these courses, is at the heart of much of modern mathematics and forms essential background for the understanding of these subjects and their applications in other parts of analysis, differential equations, probability and statistics, dynamical systems, mathematical physics, computational and applied mathematics, as well as subjects such as engineering, economics, physics, and biology that depend on mathematics.

In the past, both Math 44400 and Math 44500 were required of "Pure Math" majors, whereas only Math 44400 was required of "Applied Math" majors. Beginning this year, the requirements for "Pure Math" have changed and now Math 44400 is part of a list of three courses, two of which are required, but also *2 two-course sequences* are required for the "Pure Math" major. Math 44400 and 44500 form one *two-course sequence*, but there are ten or so others that can be used. **Therefore,** some students in Math 44400 are planning to take Math 44500 and using that as a two-course sequence, but others are planning to take Math 44400 only, and choose other two-course sequences. Math 44500 will be a sequel to Math 44400 and will both fill in gaps in Math 44400 and extend the ideas of Math 44400. Many graduate programs in mathematics or statistics, and sometimes other areas, expect students to know the material in both Math 44400 and 44500. Students expecting to continue in Math 44500 may consider buying a different book from the 'official text' for Math 44400; specifically, Bartle's "Elements $\cdots$" will cover most of the two course sequence and nearly all of the material in Bartle's (and Sherbert's) "Intro $\cdots$". On the other hand, most of the Bartle and Sherbert book will be covered in Math 44400.

The official text will be

Text: *Introduction to Real Analysis*, by Robert G. Bartle and Donald R. Sherbert, Wiley, 3^{rd} Edition (2000) (ISBN 0-471-32148-6)

The topics covered in this course will be covered, although not necessarily in the same way or in the same depth, by almost any book whose title is "Real Analysis", for example, the books listed below. Those on reserve in the library are marked with "(∗)".

(∗) *Introduction to Real Analysis*, by Robert G. Bartle and Donald R. Sherbert, Wiley, 1^{st} Edition (1982) [and, later in semester, 3^{rd} Edition (2000)]

(∗) *The Elements of Real Analysis*, by Robert G. Bartle, Wiley, 2^{nd} Edition (1976)

- *Real Analysis and Foundations*, by Steven G. Krantz, 2^{nd} Edition (2004)

(∗) *Real Analysis*, by H. L. Royden, 2^{nd} Edition (1988)

- *Closer and Closer: Introducing Real Analysis*, by Carol Schumacher, (2007)
- *Introduction to Real Analysis*, by William Trench, (2002) (Available FREE(!) on web)
- *Principles of Mathematical Analysis*, by Walter Rudin, 3^{rd} Edition (1976)
- *Understanding Real Analysis*, by Paul Zorn, (2010)

Reading and writing are important skills for life, including mathematics, and they will be important in this class as well. In your homework, your writing will be graded for spelling, grammar, and clarity of exposition as well as for mathematical correctness. In tests, writing is also important, but will not be as much of a focus as for the homework.

Reading will be an active part of this course as well. Reading assignments will be given and *your reactions to the reading assignments will be due by email by 2:00pm of the day specified in the assignment.* The tests and the final exam will also include readings that are related to the material of the course and questions concerning the reading will be part of the test.

Conversations about this mathematics will help facilitate your learning in the course, so occasional attendance at office hours is encouraged. In addition, there will be periodic 'recitation' classes scheduled where questions about the course, including the homework, can be discussed (attendance is not required).

Homework, Test, Exam, and Grading Policies

In addition to the comprehensive Final Exam on December 15, there will be 2 tests during the semester.

Grades for the course will be based on the responses to the reading assignments (approximately 10%), written homework (approximately 20%), two midterm tests (approximately 20% each), and the comprehensive final examination (approximately 30%). Late homework assignments may be handed in for feedback if you wish, but they will be recorded in the gradebook as 0's and similarly, late reactions to the reading assignments will be read and may be commented on, but will be recorded as 0's. However, the lowest two homeworks and the lowest two reactions to the readings will be dropped before computing the final grades. Each homework assignment will be worth the same number of points and the reactions to the readings will be rated as *not returned or insubstantial response* (0 points), *fair* (1 point), or *good* (2 points).

In addition, there will a list of special problems, the *'A' List*, of longer, more interesting, and perhaps harder problems than those in the usual homework. Problems from this list may be handed in at any time before 5:00pm on December 15. The problems will be read and either accepted as correct or returned for rewriting and resubmission. Only one of these problems will be counted for credit but this problem will be worth the same number of points as two regular homework assignments. In order to receive an 'A' or 'A+' for the course, you must have one of the *'A' List* problems accepted as correct. Problems will be added to this list as the semester progresses.

General Academic Policies

The work you submit for homework, tests, and the final exam must be your own. During tests and the final exam, no electronic devices, including calculators and cell phones, may be powered up or even visible and no books or notes are permitted.

For homework, you will probably find it beneficial to consult with other students about the material and this kind of conversation and collaboration is encouraged. At the end of the consultation, however, each participant is expected to prepare their own summary of the discussion and their own solution to the problem or project. More information about student conduct can be found at

`http://registrar.iupui.edu/misconduct.html`

More information concerning adaptive services for learning or other disabilities at IUPUI can be found at

`http://life.iupui.edu/aes/`

The policies for this class will be those derived from IUPUI's policies on academic conduct and adaptive services.

Some Important Dates

August 23	First day of classes
September 6	Labor Day, no classes
October 17	Last day to withdraw with automatic "W" (with permission of advisor)
October 18, 19	Fall Break, no classes
November 16	Last day to withdraw (requires permission of advisor and instructor)
November 24	Thanksgiving Break!! no classes
December 13	Last day of classes
December 15	Final Exam, 6:00p – 8:00p

Approximate Course Outline

Section numbers refer to *Introduction to Real Analysis* by Bartle & Sherbert, 3^{rd} Edition.

Section	*Topic*	*Lectures*
1.1,2,3	Preliminaries	2
2.1,2	Real numbers as an ordered field	1
2.3,4,5	Completeness of the real numbers	3
3.1,2,3	Sequences & their limits, role of monotonicity	2
3.4,5,6	Bolzano-Weierstrass Thm., Cauchy criterion, divergence	2
	Midterm Test I (late September/early October)	
4.1,2,3	Limits of functions	2
5.1,2,3	Continuous functions	2
5.4,6	Uniform continuity, monotone, inverse functions	4
6.1	Derivatives	1
	Midterm Test II (early November)	
6.2,3,4	Mean Value Theorem, L'Hôpital's & Taylor's Theorems	3
7.1,2,3	Riemann integration	3
8.1,2	Sequences of functions	2
(3.7, 9.?)	Infinite series (if time permits)	(?)
	Review	1
	Final Exam (Wednesday, December 15, 6:00 – 9:00p)	